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APPROACH TO EQUILIBRIUM IN THE CNO BI-CYCLE*

GEORGEANNE R. CAUGHLAN†

Montana State College, Bozeman, Montana

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ABSTRACT

Differential equations of the CNO bi-cycle are solved to obtain the abundances of carbon, nitrogen, and oxygen nuclei as functions of protons consumed per initial nucleus for hydrogen burning in the temperature range 10-50 × 10^6 °K. General features of the curves of abundance ratios for C^{12}, C^{13}, N^{14}, N^{15}, and O^{16} nuclei as functions of protons consumed are found to be insensitive to the temperature at which the reactions take place. The resonance at 65 keV in the $O^{17}(p,\alpha)$ reaction is responsible for the temperature dependence of ratios involving O^{17} nuclei. Analysis is made of the effect of convection with microscopic mixing on CN material passing through a hydrogen-burning zone. If a simple linear relationship or a sinusoidal relationship is assumed for the time dependence of temperature, the ratios of abundances attained at the surface in a given time are the same whether the time is spent in a single transit to the surface or in several transits back and forth from the bottom of the hydrogen-burning zone to the surface of the star. It is shown that the C^{13} to C^{12} ratio observed in typical carbon stars can be explained on the basis of the CNO bi-cycle if the time spent in the process is at least one half the mean lifetime of C^{12} for proton capture, but that in order to explain the ratio of nitrogen to oxygen suggested by astronomers for these stars, it is necessary to assume that the time spent by the nuclei at the site of hydrogen burning must be no longer than the mean lifetime of C^{12} for proton capture. In the appendix a table is given of contributions of several resonant levels in the compound nuclei involved in the proton-capture reactions of C^{12}, C^{13}, and N^{14}. These contributions are presented in the form of equations for the temperature dependence of the mean lifetimes of the reactions.

author

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† National Science Foundation Science Faculty Fellow at the California Institute of Technology, summers of 1961, 1962, and 1963. Research participant at the University of Washington, summer of 1964.

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I. INTRODUCTION

The ratio of C^{13} to C^{12} of approximately 1 to 4 observed in typical carbon stars (Aller 1961; McKellar 1948 and 1960; Bidelman 1956; Climenhaga 1960; Burbidge, Burbidge, Fowler, and Hoyle 1957; Wyller 1957 and 1960) suggests that such stars have experienced hydrogen burning through the CNO bi-cycle during their evolution. The ratio of C^{13} to C^{12} at equilibrium in the CNO bi-cycle is quite independent of the temperature at which this hydrogen burning has occurred (Caughlan and Fowler 1962). Although the ratio of nitrogen to carbon in carbon stars is not known, there is no reason to believe it is different from that in normal stars in which the abundance of nitrogen is slightly less than that of carbon (Greenstein 1962, Keenan 1960, Whitford 1961, Bidelman 1964). If the CNO bi-cycle has been active long enough to attain equilibrium, the abundance of N^{14} should be very large compared to the abundances of all the other CNO nuclei. The ratio of nitrogen to carbon at equilibrium depends on the temperature at which the bi-cycle has been active and ranges from 386 at $T_6 = 10$ to 12 at $T_6 = 100$ (Caughlan and Fowler 1962). In accordance with the usual notation, T_6 is the temperature in 10^6 °K. In search of an explanation for the probable lack of enhancement of nitrogen in carbon stars, analysis is made of the abundances of carbon, nitrogen, and oxygen nuclei as functions of protons consumed per initial nucleus in the approach to equilibrium, and it is found that if the bi-cycle has operated for a time short compared to the mean lifetime of C^{12} for proton capture, a ratio of nitrogen to carbon of less than or equal to one can be explained. The ratio of C^{13} to C^{12} varies only slightly from its equilibrium value throughout the approach to equilibrium, so the short time required to explain the N-C ratio does not affect the explanation of the C^{13} to C^{12} ratio. The number of protons

consumed per initial nucleus to attain these ratios is insensitive to the temperature at which the reactions take place in the temperature range $T_6 =$ 10-50.

It is not the author's intent to suggest stellar models in which the nuclei are brought to the surface of the star where they may be observed by spectrographic means, but let us consider the possibility that such mixing may occur by convection of nuclei through a hydrogen-burning zone and a cool envelope to the surface, and assume that the time spent at the site of hydrogen burning may be short compared to the time required to attain equilibrium in the CN cycle. Although complete microscopic mixing is assumed in this analysis, this is not always necessarily the case. It is possible that C^{12} and O^{16} may come directly to the surface without experiencing nuclear reactions. This corresponds to a situation in which no protons are consumed by the CNO nuclei. The fact that the number of protons consumed per initial nucleus is a more fundamental variable than time is emphasized by this reasoning. That is, if the surface of the star is composed of a mixture of material that has experienced reactions with different numbers of protons, the important variable is not time, but the number of protons consumed per initial nucleus. In the general case, an average over "exposures" in numbers of protons consumed should be made.

Investigation of the effect of a simple linear dependence of temperature on time for the CN cycle in several possible transits of CN nuclei back and forth through the star shows that for this simple time dependence of temperature, the final ratios of abundances depend only on the total convection time and not on the number of trips made in that time. An analysis is also given of the effect of a sinusoidal time dependence of temperature and its effect

on the carbon and nitrogen abundances. In the sinusoidal case, the CN cycle activity proceeds more rapidly than in the linear case, but again, the abundances at the surface depend only on the time of convection and not on the number of trips through the star in that time.

II. THE DIFFERENTIAL EQUATIONS

The reactions involved in the CNO bi-cycle are shown in Table 1 for temperatures less than or of the order 10^8 °K. The time rate of change of abundance of a given nucleus is determined by the reaction rates of the reactions that deplete or augment that nucleus. (In all cases, it is assumed that beta-decay lifetimes are short compared to times for proton captures.) For example, the time rate of change of abundance of C^{12} is given by

$$\frac{dC^{12}}{dt} = - p\lambda_{p\gamma}^{C12}C^{12} + p\lambda_{p\alpha}^{N15}N^{15} , \tag{1}$$

where $p\lambda_{p\gamma}^{C12}C^{12}$ is the mean reaction rate of the $C^{12}(p,\gamma)N^{13}$ reaction, the symbols C^{12} and N^{15} represent the abundances of C^{12} and N^{15} respectively, in numbers of nuclei per cm^3, p is the proton density, λ_{jk}^{i} is the average of the cross-section multiplied by the velocity $(\langle\sigma v\rangle_{ij})$ in $cm^3\ sec^{-1}$ for the interaction of particles i and j, and the subscript k indicates the lighter of the two nuclei (or the radiation) emitted in the interaction of particles i and j. Similar equations can be written for the rates of change of the other five nuclei, C^{13}, N^{14}, N^{15}, O^{16}, and O^{17}. In this analysis, the $O^{17}(p,\gamma)F^{18}$ reaction is considered negligible compared to that of the $O^{17}(p,\alpha)N^{14}$ reaction.

It is convenient to deal with equations in which much of the time dependence of quantities involved is absorbed in a new independent variable ψ, which

is dimensionless and defined by

$$\frac{\tau^o}{p} p_o \frac{d}{dt} = \frac{d}{d\psi} \quad , \tag{2}$$

where τ^o is the characteristic mean lifetime of the CNO bi-cycle defined in the notation of Caughlan and Fowler (1962)

$$\tau = \alpha(\tau_{12} + \tau_{13}) + \tau_{14} + \tau_{15} + \gamma(\tau_{16} + \tau_{17}) + \tau^{\beta} \tag{3}$$

and evaluated at $p = p_o$, and $p_o = p(t = 0)$ is the initial proton density. τ is the mean lifetime of the ith nucleus for proton capture, τ^{β} is an overall mean lifetime of 1048 sec for beta-decay processes, and is so short compared to the proton-capture lifetimes that it can be neglected at temperatures below $T_6 = 50$. The parameter $\alpha \sim 1$ is the relative alpha-particle reaction rate for $N^{15} + p$, and $\gamma \ll 1$ is the relative gamma-reaction rate for $N^{15} + p$, $(\alpha + \gamma \equiv 1)$.

The reciprocal of the initial mean lifetime of C^{12} for proton capture is

$$\frac{1}{\tau^o_{12}} = p_o \lambda^{C12}_{p\gamma} \quad , \tag{4}$$

and the reciprocal of the initial mean lifetime of N^{15} for proton capture and its relationship to the mean lifetime $\tau^o_{15,p\alpha}$ from the $N^{15}(p,\alpha)C^{12}$ reaction is

$$\frac{1}{\tau^o_{15}} = p_o \lambda^{N15}_{p\gamma} + p_o \lambda^{N15}_{p\alpha} = \frac{1}{\alpha\tau^o_{15,p\alpha}} \quad . \tag{5}$$

Combining equations (4) and (5) with equation (1) multiplied by $\tau^o p_o/p$ yields for the ψ rate of change of C^{12}

$$\frac{dC^{12}}{d\psi} = - \frac{\tau^o}{\tau^o_{12}} C^{12} + \alpha \frac{\tau^o}{\tau^o_{15}} N^{15} \quad . \tag{6}$$

Similar combinations of equations yield for the differential equations for the other five nuclei

$$\frac{dC^{13}}{d\psi} = -\frac{\tau^o}{\tau^o_{13}} C^{13} + \frac{\tau^o}{\tau^o_{12}} C^{12} \quad , \tag{7}$$

$$\frac{dN^{14}}{d\psi} = -\frac{\tau^o}{\tau^o_{14}} N^{14} + \frac{\tau^o}{\tau^o_{13}} C^{13} + \frac{\tau^o}{\tau^o_{17}} O^{17} \quad , \tag{8}$$

$$\frac{dN^{15}}{d\psi} = -\frac{\tau^o}{\tau^o_{15}} N^{15} + \frac{\tau^o}{\tau^o_{14}} N^{14} \quad , \tag{9}$$

$$\frac{dO^{16}}{d\psi} = -\frac{\tau^o}{\tau^o_{16}} O^{16} + \gamma \frac{\tau^o}{\tau^o_{15}} N^{15} \quad , \tag{10}$$

$$\frac{dO^{17}}{d\psi} = -\frac{\tau^o}{\tau^o_{17}} O^{17} + \frac{\tau^o}{\tau^o_{16}} O^{16} \quad . \tag{11}$$

The cyclic nature of the series of CNO reactions requires conservation of CNO nuclei. This conservation is mathematically inherent in equations (6) through (11) as one can see by summing the equations to obtain

$$\frac{d}{d\psi} (C^{12} + C^{13} + N^{14} + N^{15} + O^{16} + O^{17}) = 0 \quad . \tag{12}$$

This relationship can be expressed by the statement that the sum of abundances of all CNO nuclei at any time t (or at any value of the variable ψ) is equal to the initial density of CNO nuclei, I_o.

$$C^{12} + C^{13} + N^{14} + N^{15} + O^{16} + O^{17} = I_o \quad . \tag{13}$$

At equilibrium, the nuclei have attained their constant equilibrium values

C_e^{12}, C_e^{13}, etc., and the derivative of each abundance must vanish. Setting $(dC^{12}/d\psi)_{\text{equilibrium}} = 0$ in equation (6) yields a relationship for the equilibrium value of N^{15} in terms of C_e^{12}:

$$N_e^{15} = \frac{\tau_{15}^{o}}{\alpha\tau_{12}^{o}} C_e^{12} \quad , \tag{14}$$

and similar use of the other equations gives relationships for the equilibrium abundances of the other nuclei in terms of C_e^{12}. Substitution of these relationships into equation (13) expressed in terms of equilibrium values yields

$$\frac{C_e^{12}}{\alpha\tau_{12}^{o}} \left[\alpha(\tau_{12}^{o} + \tau_{13}^{o}) + \tau_{14}^{o} + \tau_{15}^{o} + \gamma(\tau_{16}^{o} + \tau_{17}^{o})\right] = I_o \quad , \tag{15}$$

which, according to equation (3) (neglecting τ^{β}) is

$$C_e^{12} = \frac{\alpha\tau_{12}^{o}}{\tau^{o}} I_o \quad . \tag{16}$$

The equilibrium abundances and the relative reaction rates, α and γ, are only very slowly varying functions of the temperature, whereas the reciprocal mean lifetimes ($p\lambda_{p\gamma}^{C12}$, etc.) that are the coefficients in equation (1) are more rapidly varying functions of temperature (Caughlan and Fowler 1962). In order to write the differential equations in a convenient form in which the coefficients are quite insensitive to temperature, let us define $\omega_{12} = \alpha I_o/C_e^{12}$, $\omega_{13} = \alpha I_o/C_e^{13}$, $\omega_{14} = I_o/N_e^{14}$, $\omega_{15} = I_o/N_e^{15}$, $\omega_{16} = \gamma I_o/O_e^{16}$, and $\omega_{17} = \gamma I_o/O_e^{17}$, and the differential equations become

$$\frac{dC^{12}}{d\psi} = -\omega_{12} C^{12} + \alpha\, \omega_{15} N^{15} \quad , \tag{17}$$

$$\frac{dC^{13}}{d\psi} = -\omega_{13} C^{13} + \omega_{12} C^{12} \quad , \tag{18}$$

$$\frac{dN^{14}}{d\psi} = -\omega_{14}N^{14} + \omega_{13}C^{13} + \omega_{17}O^{17} \quad , \tag{19}$$

$$\frac{dN^{15}}{d\psi} = -\omega_{15}N^{15} + \omega_{14}N^{14} \quad , \tag{20}$$

$$\frac{dO^{16}}{d\psi} = -\omega_{16}O^{16} + \gamma\omega_{15}N^{15} \quad , \tag{21}$$

$$\frac{dO^{17}}{d\psi} = -\omega_{17}O^{17} + \omega_{16}O^{16} \quad . \tag{22}$$

To solve this set of simultaneous equations, assumption is made of solutions of the form

$$C^{12}(\psi) = \sum_i C_i^{12} \exp(-\mu_i\psi), \; C^{13}(\psi) = \sum_i C_i^{13} \exp(-\mu_i\psi), \text{ etc.} \tag{23}$$

where the summation index i runs from 12 to 17. Substitutions of these assumed solutions into equations (17) through (22) yield a secular equation in the form of a sixth degree polynomial in μ_i. One root of this polynomial is zero, and, of course, the coefficients of the exponential terms containing this root are just the equilibrium abundances C_e^{12}, C_e^{13}, etc. Because of the long lifetime of N^{14} to the $N^{14}(p,\gamma)O^{15}$ reaction compared to the other nuclear lifetimes, $N_e^{14} \sim I_o$ and $\omega_{14} \sim 1$. It is thus natural to designate the root equal to zero as $\mu_{14} = 0$, and hence $C_{14}^{12} = C_e^{12}$, $C_{14}^{13} = C_e^{13}$, etc. The equilibrium abundances of Caughlan and Fowler (1962) have been used in a double precision computer program to solve the secular equation for the other five roots. In Table 2, values are presented for ω_i and μ_i for temperatures T_6 = 10, 17, 20, 30, and 50. The five non-zero μ_i are easily identified using the approximate relations $\mu_i \sim \omega_i$. At T_6 = 10, values are presented for both the simple CN cycle (in which the O^{16} and O^{17} reactions are ignored because of the very long

mean lifetimes, $\sim 10^{14}$ yr, of the O^{16} and O^{17} proton reactions at this low temperature) and the full CNO bi-cycle. In order to obtain consistent results in later calculations, it is necessary to use double precision and to keep at least 8 significant figures at this stage of the calculations.

Relationships between the coefficients of the expansion equations, C_1^{12}, etc., can be derived from equations (17) through (23). All coefficients are expressed in terms of N_1^{14}, and a double precision computer program is employed to solve the resulting six simultaneous equations in N_{12}^{14}, N_{13}^{14}, N_{15}^{14}, N_{16}^{14}, and N_{17}^{14}. These solutions depend on the initial conditions imposed on the abundances. Because the main input nuclei for the CNO bi-cycle are believed to be C^{12} and O^{16} that result from helium burning in red-giant stars, it is assumed that the initial abundances of all other CNO nuclei are zero, and the effects of different initial relative abundances of C^{12} and O^{16} are examined. In stars of approximately one solar mass, the temperature at which helium burning in the core of a red giant occurs is in the low T_8 region (T_8 is temperature in 10^8 °K) and the helium burning processes produce C^{12} by the Salpeter-Hoyle 3-α process with only small amounts of C^{12} going to O^{16} through C^{12} alpha captures (Fowler and Hoyle 1964). In massive stars, the core temperature is higher, and if helium burning has been in effect in the core of a massive star for some time, much of the C^{12} will have been processed into O^{16}. In order to explore the effects of these different initial abundances on the CNO bi-cycle, the equations are solved for the following three sets of initial conditions, all of which imply $C^{13}(0) = N^{14}(0) = N^{15}(0) = O^{17}(0) = 0$:

$$C^{12}(0) = I_o,\ O^{16}(0) = 0 \quad , \tag{24a}$$

$$C^{12}(0) = 0,\ O^{16}(0) = I_o \quad , \tag{24b}$$

$$C^{12}(0) = O^{16}(0) = 0.5\ I_o \quad . \tag{24c}$$

One should expect conditions (24a) for low mass stars ($M < 3\ M_\odot$), (24b) for massive stars ($M > 30\ M_\odot$), and (24c) for stars of intermediate mass ($M \sim 10\ M_\odot$). The case (24c) serves as a check on the other two solutions and to show that any initial ratio of C^{12} to O^{16} can be solved as a combination of the first two cases. If N_a is the abundance of nucleus N in case (24a) and N_b is its abundance in case (24b) for a given value of ψ, and if the initial abundance of C^{12} is xI_o, the abundance N for this initial ratio case is $xN_a + (1 - x)N_b$.

III. ABUNDANCES AS FUNCTIONS OF PROTONS CONSUMED

The rates of change of He^4 nuclei and of protons as alpha-particles are produced and protons are consumed in the CNO bi-cycle are

$$\frac{dHe^4}{d\psi} = \alpha\omega_{15}N^{15} + \omega_{17}O^{17} , \tag{25}$$

$$\frac{dp}{d\psi} = -\omega_{12}C^{12} - \omega_{13}C^{13} - \omega_{14}N^{14} - \omega_{15}N^{15} - \omega_{16}O^{16} - \omega_{17}O^{17} . \tag{26}$$

In addition to the above-mentioned conservation of CNO (seed) nuclei, there must also be conservation of nucleons in the reactions so that

$$\Delta p + 4\Delta He^4 + \Delta C^{13} + 2\Delta N^{14} + 3\Delta N^{15} + 4\Delta O^{16} + 5\Delta O^{17} = 0. \tag{27}$$

if $C^{12}(0) = I_o$ or a similar equation if $O^{16}(0) = I_o$. Either imposition of this requirement combined with equation (13) or algebraic manipulation of equations (25), (26), and (17) through (22) yields for the protons consumed per initial seed nucleus

$$\frac{p_o - p}{I_o} = \sum_{4,12}^{17} A_i\,(a - a_o)_i , \tag{28}$$

where $(a - a_o)_i$ is the difference between the abundance of the ith nucleus as a function of ψ and its original abundance and A_i is the atomic mass number of the ith nucleus. The running index, i, is summed on 4 for He^4 and on 12 through 17 for the CNO nuclei.

Tables 3, 4, and 5 show the relative abundances and ratios of abundances of the CNO nuclei as functions of protons consumed per initial nucleus at $T_6 = 20$ for the three sets of initial conditions given by equations (24a,b,c). Graphs of relative abundances as functions of protons consumed per initial nucleus for the three cases at $T_6 = 20$ are presented in Figures 1, 2, and 3. In Figures 4 and 5, the ratios of abundances as functions of protons consumed per initial nucleus for the $C^{12}(0) = O^{16}(0) = 0.5\ I_o$ case at $T_6 = 20$ and at $T_6 = 50$ are presented. To show the time dependences of the functions, time is given in the tables in the form $t(\rho x_H)_o/100$ and in the form either t/τ^o_{12} or t/τ^o_{16}, whichever is pertinent. Also, a secondary abscissa of time in the form $t(\rho x_H)_o/100$ is plotted along the top of Figures 1, 2, and 3, to show at least order of magnitude dependences of abundances and ratios of abundances on the time. Because of the complicated relationship between protons consumed per initial nucleus and time, interpolation between values of time given in the figures is difficult, and any reader who wishes to find a more exact time for a given value of one of the functions is advised to consult the tables where smaller increments in time are presented than is possible in the graphs. Note that time relationships in the form presented do not take into account the depletion of protons as time progresses. This is not a serious difficulty, at least in the early stages of the approach to equilibrium. The time scale employed in Figures 4 and 5 is t/τ^o_{12} or t/τ^o_{16} to emphasize the dependences of values of the ratios of abundances on the pertinent mean lifetimes.

The general features of the curves show that if any C^{12} is present initially, the CN part of the bi-cycle dominates the early stages of the approach to equilibrium, a reflection of the fact that the mean lifetime of O^{16} is large compared to the mean lifetimes of C^{12} and C^{13} for proton capture. The CN cycle comes into equilibrium when approximately 2.3 protons have been consumed per initial nucleus if $C^{12}(0) = I_o$ as shown in Figure 1. The effect of the NO part of the bi-cycle begins to show shortly after the initial "equilibrium" of the CN nuclei has occurred. This NO effect is not evident in Figure 1 because it is too slight to be seen to three place figures; however, if five digits are kept in the N^{14} abundance, the onset of NO activity can be seen in a small hump in the N^{14} curve when approximately 2.4 protons have been consumed per initial nucleus in this case. Figure 2 shows the lack of initial CN activity expected in the $O^{16}(0) = I_o$ case, the C^{12} and C^{13} build-up due to NO activity, and the late approach of all CNO nuclei to equilibrium. It is interesting to note that Figure 3 shows the combined effects seen in Figures 1 and 2. The initial CN "equilibrium" due to original C^{12} capturing protons is seen at 1.5 protons consumed per initial nucleus, then after 3 protons have been consumed, the CN nuclei experience increases due to the NO activity; and the actual equilibrium of all CNO nuclei is late, when more than 10^3 protons have been consumed per initial nucleus.

Calculations and graphs (not shown) of the $C^{12}(0) = 0.1\ I_o$, $O^{16}(0) = 0.9\ I_o$ case show the initial CN "equilibrium" when approximately 0.2 protons have been consumed per initial nucleus, a pronounced evidence of NO activity after approximately 0.6 protons have been consumed, and final equilibrium of all CNO nuclei after more than 10^3 protons have been consumed.

A particularly interesting feature of the curves of ratios of abundances

(Figures 4 and 5) is the fact that the C^{13}/C^{12} ratio rises rapidly early in the approach to equilibrium to a maximum value ~ 0.33 that is only slightly greater than the equilibrium ratio and then reduces to the equilibrium value of 0.25 by the time of CN equilibrium. Thus, the C^{13}/C^{12} ratio, which is quite insensitive to temperature change at equilibrium, is also not very sensitive to time spent in hydrogen burning at a given temperature once one proton has been consumed per initial nucleus.

The ratio N/C also rises rapidly in the approach to equilibrium to attain very nearly its equilibrium value with nitrogen dominating the CNO nuclei by the time of CN equilibrium.

The effect of the NO part of the bi-cycle is obvious in the C/O and N/O ratios. C/O drops to a minimum as CN "equilibrium" sets in and then rises as the NO part of the bi-cycle becomes effective. N/O rises rapidly due to CN activity, levels off at CN "equilibrium", and then rises again due to NO activity.

Comparison of Figures 4 and 5 shows that, with the exception of the O^{17}/O^{16} curve, the main features of the curves are quite insensitive to temperature. The equilibrium values attained are different, but the points of maxima, minima, or leveling off occur at approximately the same numbers of protons consumed per initial nucleus at $T_6 = 20$ and at $T_6 = 50$. Results of calculations at $T_6 = 10$, 17, 20, 30, and 50 establish the fact that this lack of sensitivity to temperature of the main features of the curves (of ratios of abundances as functions of protons consumed per initial nucleus) holds throughout the temperature range $T_6 = 10$-50. Pertinent points in the approach to equilibrium for the $C^{12}(0) = I_o$ case are displayed for each temperature in Table 6A. The four points considered are: 1) the C^{13}/C^{12} ratio reaches a value equal to its equilibrium value as it rises to a maximum, 2) N/C = 1.00, 3) C^{13}/C^{12} is at

its maximum value, and 4) CN "equilibrium" is established. The dramatic difference in the O^{17}/O^{16} ratio at $T_6 = 20$ and at $T_6 = 50$ seen in Figures 4 and 5 is due to the resonance at 65 keV in the $O^{17}(p,\alpha)N^{14}$ reaction and the resulting small mean lifetime of O^{17} at the higher temperature (Brown 1962). This has the effect of bringing O^{17}/O^{16} to a constant value very early in the approach to equilibrium and also of making this equilibrium value much smaller at the higher temperature. The fact that the ratio of the mean lifetime of O^{16} to that of C^{12} is

smaller at $T_6 = 50$ than at $T_6 = 20$ does effect the C/O and N/O curves in that the NO activity is apparent at fewer protons consumed at the higher temperature and that the level at CN "equilibrium" is of shorter duration.

It is of particular interest to examine the time scales for certain aspects of the C^{13}/C^{12}, N/C, and N/O ratios at the two temperatures. If the original proton density, p_o, is very large compared to the original seed nuclei density, I_o, the change in proton density early in the approach to equilibrium is negligible, and the time given in the next-to-the last column of Tables 3, 4, and 5 (assuming $(\rho x_H)_o = 100$ gm cm^{-3}) is a good measure of the time elapsed since the onset of hydrogen burning in the CNO bi-cycle. In Table 6B are presented the time for C^{13}/C^{12} to rise to 0.25 in the rise to maximum, the time for N/C to rise to 1.0, the time at which C^{13}/C^{12} is at a maximum, the time at which CN equilibrium occurs, and the time for N/O to attain the value unity for the case of $C^{12}(0) = O^{16}(0) = 0.5\ I_o$ at $T_6 = 20$ and at $T_6 = 50$. In each case, the time is given in years, then as the ratio of the time to the mean lifetime of C^{12} or of O^{16}, whichever is pertinent. These time ratios are also used as time scales in Figures 4, 5, and 6. Figure 6 shows the very small variation of C^{13}/C^{12} as a function of the more rapidly changing N/C ratio. The tables and graphs show that if the time spent in hydrogen burning is as long as one-half the mean lifetime of C^{12}, the ratio C^{13}/C^{12} is in the range 0.2 to 0.3 which is compatible with the observed values of this ratio in typical carbon stars. In a little less than the mean lifetime of C^{12}, the abundance of nitrogen rises to a value equal to that of carbon, and at this time the C^{13}/C^{12} ratio is 0.31 at either temperature. The C^{13}/C^{12} ratio reaches a maximum value in a little less than 2 C^{12} mean lifetimes, and CN "equilibrium" occurs in a little less than 10 C^{12} mean

lifetimes. For nitrogen to attain the same abundance as oxygen in this particular case, 0.02 of the mean lifetime of O^{16} must have elapsed. Although these calculations are based on the special case of C^{12} and O^{16} being equally abundant in the original material, the times for the first four phenomena are the same, independent of the original ratio of C^{12} to O^{16} provided <u>some</u> C^{12} is present in the original material. The time for N/O to equal unity is quite sensitive to the original ratio of C^{12} to O^{16} because of the more rapid formation of N^{14} through the CN cycle than through the NO part of the bi-cycle.

Astronomers observe values of from 1/5 to 1/3 for the C^{13}/C^{12} ratio in typical carbon stars and assume that nitrogen is no more abundant than carbon in these stars. The data presented in Table 6 show that it is possible to explain these ratios on the basis of hydrogen burning in the CNO bi-cycle if the original material contains some C^{12} and if the time spent in hydrogen burning is less than the mean lifetime of C^{12} for proton capture. This assumes the abundance of nitrogen in the original material is negligible compared to that of carbon. If some nitrogen is present originally, the time for N/C to reach unity is, of course, decreased; for example, if N^{14} is one-fourth as abundant as C^{12} in the original material, N/C will reach unity in approximately 6/10 the mean lifetime of C^{12}.

Further emphasis of the small variation of the C^{13}/C^{12} ratio compared to the N/C ratio is made in Figure 6 where a plot of these ratios shows C^{13}/C^{12} varying only between 0 and 0.33 while N/C rises rapidly to a value of 94.4 when CN equilibrium is reached for $T_6 = 20$, $C^{12}(0) = I_o$. The time scale given in Table 6A is shown as abscissae along the top of Figure 6.

IV. CN NUCLEI IN A CONVECTIVE MEDIUM

If a simple linear relationship is assumed between time and the temperature of the medium in which the nuclei are moving, then

$$T = T_o (1 - t/t_f) \quad , \tag{29}$$

where T_o is the temperature at zero time (i.e., the temperature at the bottom of the hydrogen-burning shell) and t_f is a "final" time in which the nuclei reach the surface of the star. A polytropic relationship between density and temperature

$$\rho = \rho_o \left(\frac{T}{T_o}\right)^n \tag{30}$$

is assumed, where ρ_o is the density at the bottom of the hydrogen-burning shell and n is the polytropic index (Schwarzschild 1958). The dependence of proton density on time is due to two effects, first the change in density of the medium, which will cause a time variation of

$$p = p_o (1 - t/t_f)^n \quad , \tag{31}$$

where p_o is the proton density at the bottom of the hydrogen-burning shell, and secondly the change in relative abundance of hydrogen due to proton consumption in the CN cycle. The latter effect is small compared to the former so, for purposes of integration, the time dependence of proton density is assumed to be given by equation (31), and the secondary time effect of proton consumption on the abundance of hydrogen is determined by solution of the reaction-rate equations.

The changes in temperature and density as the nuclei move outward will, of course, affect the mean lifetimes of the proton capture reactions. An

inverse power relationship

$$\tau_i^o = \tau_i^{oo}\,(T_o/T)^{m_i} = \tau_i^{oo}/(1 - t/t_f)^{m_i} \qquad (32)$$

is assumed for the time dependence of the mean lifetime of the ith nucleus. τ_i^{oo} represents the mean lifetime of the ith nucleus under conditions prevailing at the bottom of the hydrogen-burning shell at t = 0.

A further simplification is imposed on the problem by assuming that the time of hydrogen burning is long enough that C^{13} and N^{15} are in equilibrium, and in addition that the abundances of C^{13} and N^{15} are negligible compared to those of C^{12} and N^{14}. The conservation of seed nuclei then reduces to

$$C^{12}(t) + N^{14}(t) = C^{12}(0) \qquad (33)$$

since C^{12} is assumed to be the only CN nucleus present originally.

If one assumes that the temperature dependence of the mean lifetime of N^{14} is the same as that for C^{12} (i.e., $m_{12} = m_{14} = m$), it is possible to analytically integrate the reaction equations to obtain interesting information relative to this simple case. Imposition of the time dependence of proton density and mean lifetimes given by equations (31) and (32) yields for the differential equations for the abundances of C^{12}, N^{14}, and p

$$\frac{dC^{12}}{dt} = (1 - t/t_f)^{m+n}\left[- C^{12}/\tau_{12}^{oo} + N^{14}/\tau_{14}^{oo}\right], \qquad (34)$$

$$\frac{dN^{14}}{dt} = (1 - t/t_f)^{m+n}\left[- N^{14}/\tau_{14}^{oo} + C^{12}/\tau_{12}^{oo}\right], \qquad (35)$$

$$\frac{dp}{dt} = -2(1 - t/t_f)^{m+n}\left[C^{12}/\tau_{12}^{oo} + N^{14}/\tau_{14}^{oo}\right]. \qquad (36)$$

Equation (36) gives the time rate of consumption of protons mentioned above as

a smaller time effect than that due to the change of density of the medium. Solution of these equations using the conservation of seed nuclei condition of equation (33) yields

$$C^{12}(t) = \frac{C^{12}(0)}{\tau^{00}} \left\{ \tau_{12}^{00} + \tau_{14}^{00} \exp\left[- \frac{\tau^{00}\, t_f}{\tau_{12}^{00}\, \tau_{14}^{00}\, r} \left(1 - [1 - t/t_f]^r \right) \right] \right\}, \quad (37)$$

$$N^{14}(t) = \frac{C^{12}(0)}{\tau^{00}} \left\{ \tau_{14}^{00} - \tau_{14}^{00} \exp\left[- \frac{\tau^{00}\, t_f}{\tau_{12}^{00}\, \tau_{14}^{00}\, r} \left(1 - [1 - t/t_f]^r \right) \right] \right\}, \quad (38)$$

$$\Delta p = p(0) - p(t) = \frac{C^{12}(0)}{\tau^{00}} \left\{ \frac{4t_f}{r} \left[1 - (1 - t/t_f)^r \right] \right.$$

$$\left. + \frac{2\, \tau_{14}^{00} \left(\tau_{14}^{00} - \tau_{12}^{00} \right)}{\tau^{00}} \left[1 - \exp\left(- \frac{\tau^{00}\, t_f}{\tau_{12}^{00}\, \tau_{14}^{00}\, r} \left[1 - (1 - t/t_f)^r \right] \right) \right] \right\}, \quad (39)$$

where $\tau^{00} = \tau_{12}^{00} + \tau_{14}^{00}$ and $r = m + n + 1$.

The effect of nuclei moving by convection back and forth between the bottom of the hydrogen-burning shell and the surface of the star is determined by solving equations similar to equations (34) through (36) derived with the condition that as the nuclei move back toward the center

$$T = T_o\, (t/t_{f1}) \quad (40)$$

where t_{f1} is the time to move from the surface back to the center, and it is assumed that convective velocities are of the same magnitude whether motion is inward or outward so that $t_{f1} = t_f$. Again, T_o is the temperature at the bottom of the hydrogen-burning zone. The expression for protons consumed as

a function of time on the return trip is

$$\Delta p(t_k) = \frac{C^{12}(0)}{\tau^{\infty}} \left\{ \frac{4t_f}{r} (t/t_f)^r \right.$$

$$\left. + \frac{2\,\tau_{14}^{\infty}\left(\tau_{14}^{\infty} - \tau_{12}^{\infty}\right)}{\tau^{\infty}} \exp\left(-\frac{\tau^{\infty}\, t_f}{\tau_{12}^{\infty}\,\tau_{14}^{\infty}\, r}\right)\left[1 - \exp\left(-\frac{\tau^{\infty}\, t_f}{\tau_{12}^{\infty}\,\tau_{14}^{\infty}\, r}\right)\left(\frac{t}{t_f}\right)^r\right]\right\} \quad (41)$$

Extrapolation of these considerations to repeated mixing in and out in a seesaw fashion assuming immediate return at bottom or top of convective zone shows that for the kth trip (where k = 0 for the first trip out, k = 1 for the first trip in, etc.), the expression for protons consumed per initial seed nucleus as a function of time is

$$\frac{\Delta p}{C^{12}(0)} = \frac{1}{\tau^{\infty}}\left\{\frac{4t_f}{r}A + \frac{2\,\tau_{14}^{\infty}\left(\tau_{14}^{\infty} - \tau_{12}^{\infty}\right)}{\tau^{\infty}} \exp\left(-\frac{k\,\tau^{\infty}\, t_f}{\tau_{12}^{\infty}\,\tau_{14}^{\infty}\, r}\right)\left[1 - \exp\left(-\frac{\tau^{\infty}\, t_f}{\tau_{12}^{\infty}\,\tau_{14}^{\infty}\, r}A\right)\right]\right\} \quad (42)$$

where t is measured from the time of leaving the center for trips out and of leaving the surface for return trips, and $A = 1 - (1 - t/t_f)^r$ for mixing out and $A = (t/t_f)^r$ for mixing in. The effect of the increasing negative exponential term in decreasing the number of protons consumed in successive trips is that the number of protons consumed per initial seed nucleus is the same in a total time t_F whether that time is spent in one convective trip to the surface or in many trips back and forth and finally ending at the surface. Mathematically, this is obvious by summing terms of equation (42) evaluated at successive "final" times t_k. At each integral number of time t_f (t_k in summation), A = 1 whether convection is inward or outward. The final time t_F, when it is assumed convection stops with the nuclei at the surface, is the

time for making n + 1 trips through the star, $t_F = (n + 1)t_k$, and

$$\sum_{k=0}^{n} \Delta p(t_k) = \Delta p(t_F). \tag{43}$$

Physically, this is reasonable in that the nuclei spend the same length of time in a given temperature zone whether they move through it once slowly in one trip to the surface or several times rapidly in several trips back and forth.

Values must be assigned to the parameters T_o, r, and t_f to calculate abundances and proton consumption during convection. In order to check these results with the approach to equilibrium calculations of Section III where the nuclei remained at a given site and thus temperature, the proton consumption as a function of time up to one mean lifetime of C^{12} with $T_o = 20 \times 10^6$ °K and r = 1 (i.e., no dependence of density or mean lifetimes on time) was made. The results show 1.3 protons consumed per initial nucleus in $t_f = 6.61 \times 10^3$ yr which is in reasonable agreement with the 1.2 protons of the approach to equilibrium calculations especially when one considers the fact that the unrealistic assumption of C^{13} being in equilibrium even in the early stages of the approach to equilibrium, made in the convection analysis, leads to a consumption of more protons than occurs in the more rigorous analysis of Section III. Reasonable values of m and n suggest values of r in the range 14 to 20. A plot in Figure 7 of protons consumed per initial nucleus as a function of the dimensionless time scale t/t_f, where t_f is assigned the value of one mean lifetime of C^{12} for proton capture at 20×10^6 °K, for r = 14, 16, 18, and 20, shows that in this range, the power dependence does not strongly affect the protons consumed, and that if convection has taken place for one mean lifetime of C^{12} (6.61×10^3 yr

at 20×10^6 °K), approximately 0.1 proton has been consumed per initial nucleus. Calculations for $T_o = 50 \times 10^6$ °K show little difference in a consumption of protons if the time is one mean lifetime of C^{12} (1.86×10^{-2} yr) at that temperature; in fact, the plot would be indistinguishable from Figure 7. The consumption of only 0.1 proton per initial nucleus in convection effective for one mean lifetime of C^{12} would indicate that CNO nuclei should show abundances reflecting CNO bi-cycle activity in the very early stages of the approach to equilibrium with little C^{13} or N^{14} formed. Further calculations show that in this seesaw type of convection, in order to attain the approximately one proton consumed per initial nucleus stage at which the approximate ratios of unity for N/C and 0.25 for C^{13}/C^{12} occur in the bi-cycle approach to equilibrium, convection must take place for longer than ten mean lifetimes of C^{12} for proton capture ($> 7 \times 10^4$ yr if $T_o = 20 \times 10^6$ °K or > 0.2 yr if $T_o = 50 \times 10^6$ °K). The abundances of C^{12} to be expected after several mean lifetimes of C^{12} have elapsed in convective mixing are presented in Table 7 for r = 14 and 18 and for T_o = 20×10^6 and 50×10^6 °K.

Perhaps a more realistic picture of convection is one in which the material moves through a medium in such a way that its temperature varies sinusoidally with time. Solution of reaction-rate equations for C^{12} and N^{14} under the condition that

$$T = (T_o/2)(1 + \cos \omega t) = T_o \cos^2 (\omega t/2) \tag{44}$$

yields

$$C^{12}(t)/C^{12}(0) = \frac{1}{\tau^{oo}} \left\{ \tau_{12}^{oo} + \tau_{14}^{oo} \exp \left[\frac{-2\, \tau^{oo}}{\omega\, \tau_{12}^{oo}\, \tau_{14}^{oo}} \left(\frac{[2(r-1)]!\ \omega t}{2^{2(r-1)+1}\ [(r-1)!]^2} + \frac{1}{2^{2(r-1)-1}} \sum_{\nu=0}^{r-2} \left\{ \frac{[2(r-1)]!}{\nu!\,[2(r-1)-\nu]!}\ \frac{\sin\,(r-1-\nu)\ \omega t}{2(r-1-\nu)} \right\} \right) \right] \right\} \tag{45}$$

Computer calculations show that depletion of C^{12} and the inherent consumption of protons is slightly more rapid in this case than in the seesaw convection case so that a convection time of approximately six or seven mean lifetimes of C^{12} suffice for the consumption of approximately one proton per initial nucleus to build C^{13} and N^{14} up to the abundances expected in typical carbon stars.

The abundances of interest in either of these cases of convection are those at the surface of the star, i.e., when $t = t_f$, convection has ceased and ωt in equations (44) and (45) is an odd integral multiple of π. Under these conditions, the equations for C^{12} reduce to the simple forms

$$C^{12}(t_f)/C^{12}(0) = \frac{1}{\tau^{oo}} \left[\tau_{12}^{oo} + \tau_{14}^{oo} \exp\left(- \frac{\tau^{oo}\, t_f}{\tau_{12}^{oo}\, \tau_{14}^{oo}\, r} \right) \right] \tag{46}$$

for the seesaw convection case, and

$$C^{12}(t_f)/C^{12}(0) = \frac{1}{\tau^{oo}} \left(\tau_{12}^{oo} + \tau_{14}^{oo} \exp\left\{ - \frac{\tau^{oo}\, t_f}{\tau_{12}^{oo}\, \tau_{14}^{oo}} \frac{[2(r-1)]!}{2^{2(r-1)}\, [(r-1)!]^2} \right\} \right) \tag{47}$$

for the sinusoidal case. Note that in the sinusoidal case also, the abundance at t_f is independent of the number of trips made through the star in that time in that it is independent of ω at t_f. The fact that C^{12} is depleted more rapidly in the latter case is obvious since $[2(r-1)]!/2^{2(r-1)}[(r-1)!]^2$ is greater than $1/r$ so that for a given value of t_f, the exponential term in equation (47) is less than that term in equation (46). Because the problems solved in the convection analysis assume C^{13} and N^{15} in equilibrium even in the early stages, the proton consumption for a given C^{12} depletion is larger in this analysis than in the rigorous approach to equilibrium calculations;

hence, an analysis of time required for the approach to equilibrium in the convective cases is more reasonable on the basis of C^{12} depletion than on proton consumption. From the approach to equilibrium calculations for $C^{12}(0) = I_o$ presented in Table 3, the abundance of C^{12} is approximately 0.4 when N and C^{13} have risen to values expected in typical carbon stars. The values of $C^{12}(t_f)$ presented in Table 7 for different values of t_f and r at $T_o = 20 \times 10^6$ and 50×10^6 ^oK illustrate the analysis of time of convection made above. Note that on the basis of C^{12} depletion, seesaw convection must take place for approximately fifteen mean lifetimes of C^{12} compared to the six or seven mean lifetimes required in sinusoidal convection to deplete C^{12} to approximately 0.4.

V. SUMMARY AND CONCLUSIONS

The ratio of the isotopic abundances of carbon, C^{13}/C^{12}, of from 0.2-0.3 observed in certain carbon stars can be explained on the basis of hydrogen burning in the CNO bi-cycle in the star whether or not the burning has progressed for a long enough time for equilibrium to be established in the CNO nuclei. In order to explain an abundance of nitrogen less than carbon, if hydrogen burning in the CNO bi-cycle is responsible for observed and expected abundances, the time of CNO bi-cycle activity must be short enough that equilibrium in the CN part of the bi-cycle has not occurred. If the activity has taken place at a single site and at constant temperature and density, the time of CNO activity must be less than one mean lifetime of C^{12} for proton capture at that temperature and density to avoid an abundance of nitrogen greater than that of carbon. If the time is at least one-half the mean lifetime of C^{12}, C^{13} will have been produced to a reasonable amount. If the CNO activity has

taken place in nuclei moving by convection through the star so that actual hydrogen burning occurs for only that small portion of the mixing time when the nuclei are passing through a hydrogen-burning zone, a time of convection of several mean lifetimes of C^{12} for proton capture at the temperature and density of the hydrogen-burning zone can explain the C^{13}/C^{12} ratios observed and the smaller abundance of nitrogen than carbon expected. The analysis can be extended to nuclei remaining at one site at which the temperature and density are varying with time. The time of activity should be expected to be greater than one mean lifetime, and less than six mean lifetimes of C^{12} for proton capture at the highest temperature and density occurring at the site.

In order to understand the lack of C^{13} in the "non-typical" carbon stars which show C^{13}/C^{12} ratios of 0.1 or less (Bidelman 1956, Climenhaga 1960, Wyller 1960, Wallerstein and Greenstein 1964), one must assume either that 1) the CNO nuclei have not undergone hydrogen burning; or that 2) C^{13} has been destroyed by some process after completion of hydrogen burning, for example, through $C^{13}(\alpha,n)O^{16}$ in helium burning; or that 3) hydrogen burning in the CNO bi-cycle lasted for a time very short compared to the mean lifetime of C^{12}. (In simple hydrogen burning in the CNO bi-cycle, the ratio of C^{13}/C^{12} exceeds 1/25 in 0.05 of a mean lifetime of C^{12} and exceeds 1/10 in less than 0.16 of a mean lifetime of C^{12} for proton capture at the temperature and density of the site of burning.) Thus, if CNO nuclei pass through a hydrogen-burning zone in convection to the surface in such stars as HD 26 and HD 201626 in which a C^{13}/C^{12} ratio of less than 1/10 is reported by Wallerstein and Greenstein (1964), the time spent in the hydrogen-burning zone must be very small compared to the mean lifetime of C^{12} for proton capture in the hydrogen-burning zone (Caughlan and Fowler 1964).

In stars in which bulk transport is effective in bringing nuclei to the surface with no microscopic mixing, unmodified C^{12} from helium burning will enhance the surface density of C^{12} with no accompanying increase in C^{13} or N^{14}. In massive stars ($> 30\ M_{\odot}$) where O^{16} is produced in helium burning to the extent that the CNO bi-cycle would find $O^{16}(0) \approx I_o$ in this analysis, the approach to equilibrium is a slow process in the CN nuclei with all of the CNO nuclei coming into equilibrium at about the same time in approximately ten mean lifetimes of O^{16}. Thus, although the C^{13}/C^{12} ratio remains very close to its equilibrium value of 0.25 from very early in the approach, C^{13} and C^{12} do not reach their equilibrium values long before N^{14} does.

In summary: the analysis of the approach to equilibrium has shown that in low or intermediate mass stars, 1) the ratio of C^{13}/C^{12} produced in CNO processes can be 0.2-0.3 with nitrogen not necessarily more abundant than carbon; 2) if C^{13}/C^{12} is much less than 0.2-0.3, then nitrogen must be much less abundant than carbon; and 3) if nitrogen is much more abundant than carbon, then C^{13}/C^{12} must be 0.25 or the nitrogen cannot be the result of CNO processes. Consideration of the CNO approach to equilibrium in massive stars shows that although the C^{13}/C^{12} ratio should be approximately 0.25 throughout the approach to equilibrium, there should be little or no enhancement of carbon by CNO processes, and the ratio of N/C should be no less than the equilibrium value of order 10-100 if the CNO bi-cycle has acted upon the O^{16} produced by helium burning in the massive stars.

When more detailed models are available for red-giant stars, and when the processes of bringing nuclei to the surface where they may be observed is understood, the analysis of this paper should provide information on the times involved in the hydrogen-burning CNO processes that seem to be

responsible for certain aspects of the abundances of CNO nuclei observed in carbon stars. Violation of any of the rules outlined in the summary paragraph above would be of interest not in casting doubt on nuclear reaction rates, but in uncovering new modes of production and/or mixing to the surface of carbon, nitrogen, and oxygen nuclei.

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APPENDIX

At temperatures in the T_8 and T_9 regions, certain levels in the compound nuclei N^{13}, N^{14}, and O^{15} become important as resonances in the capture of protons by C^{12}, C^{13}, and N^{14}. Because it is more probable that these levels will decay by proton emission than by gamma emission, the single-level resonance equation for $\Gamma_1 >> \Gamma_2$ given by B^2FH

$$\frac{1}{\tau_1(0)} = 0.49 \times 10^{13} \frac{\rho x_1}{A_1} \frac{\nu_0 \omega\Gamma_2 f_r}{(AT_6)^{3/2}} \exp(-11.61\, E_r/T_6) \quad \mathrm{sec}^{-1}$$

is appropriate. Results of calculations of the contribution of each level are presented in Table 8 in the form of log $\tau\rho x_H$ where τ is in yr. The reference column gives the source of the data for the resonance energies and for the experimentally determined values of $\omega\Gamma_\gamma$, the product of the statistical factor and the width for emission of a gamma ray with the compound nucleus of the appropriate reaction.

TABLE 1

THE CNO BI-CYCLE

($T \leq 10^8$ Degrees)

$$C^{12} + H^1 \rightarrow N^{13} + \gamma$$

$$N^{13} \rightarrow C^{13} + e^+ + \nu$$

$$C^{13} + H^1 \rightarrow N^{14} + \gamma$$

$\bar{\tau} = 870$ sec

$\log \bar{\tau}$ (yr) $= -4.56$

$$N^{14} + H^1 \rightarrow O^{15} + \gamma$$

$$O^{15} \rightarrow N^{15} + e^+ + \nu$$

$$N^{15} + H^1 \rightarrow C^{12} + He^4$$

$\bar{\tau} = 178$ sec

$\log \bar{\tau}$ (yr) $= -5.25$

or ($\sim 4 \times 10^{-4}$)

$$N^{15} + H^1 \rightarrow O^{16} + \gamma$$

$$O^{16} + H^1 \rightarrow F^{17} + \gamma$$

$$F^{17} \rightarrow O^{17} + e^+ + \nu$$

$$O^{17} + H^1 \rightarrow N^{14} + He^4$$

$\bar{\tau} = 95$ sec

$\log \bar{\tau}$ (yr) $= -5.52$

TABLE 2

DEPENDENCE OF ω_i AND μ_i ON TEMPERATURE

Cycle	CN	CNO				
T_6 =	10	10	17	20	30	50
ω_{12}	4.8484709×10^{2}	5.6151840×10^{2}	1.6795814×10^{2}	1.2229812×10^{2}	6.2558454×10^{1}	3.1731008×10^{1}
μ_{12}	4.8621980×10^{2}	5.6310750×10^{2}	1.6937624×10^{2}	1.2368672×10^{2}	6.3937940×10^{1}	3.3130454×10^{1}
ω_{13}	1.9246554×10^{3}	2.2288812×10^{3}	6.7416325×10^{2}	4.9193962×10^{2}	2.5232196×10^{2}	1.2773145×10^{2}
μ_{13}	1.9242829×10^{3}	2.2284500×10^{3}	6.7380180×10^{2}	4.9158831×10^{2}	2.5197578×10^{2}	1.2737899×10^{2}
ω_{14}	1.0026372	1.1612051	1.0572827	1.0378170	1.0337227	1.0473621
μ_{14}	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
ω_{15}	2.0735871×10^{4}	2.4038519×10^{4}	2.4831210×10^{4}	2.5476928×10^{4}	2.8711953×10^{4}	3.5010941×10^{4}
μ_{15}	2.0735873×10^{4}	2.4038522×10^{4}	2.4831210×10^{4}	2.5476928×10^{4}	2.8711953×10^{4}	3.5010941×10^{4}
ω_{16}		4.6265574×10^{-3}	1.2425449×10^{-2}	1.6336717×10^{-2}	3.1378780×10^{-2}	6.3451057×10^{-2}
μ_{16}		5.5899928×10^{-3}	1.3142904×10^{-2}	1.6776581×10^{-2}	3.1780721×10^{-2}	6.3825208×10^{-2}
ω_{17}		1.0862203×10^{-2}	3.3192585×10^{-2}	5.3483368×10^{-1}	7.9515935×10^{1}	5.8244175×10^{2}
μ_{17}		1.0411906×10^{-2}	3.2918077×10^{-2}	5.3482023×10^{-1}	7.9515935×10^{1}	5.8244175×10^{2}

TABLE 3A

RELATIVE ABUNDANCES AS FUNCTIONS OF PROTONS CONSUMED
PER INITIAL NUCLEUS FOR $T_6 = 20$, $C^{12}(0) = I_o$

$(p_o - p)/I_o$	He^4/I_o	C^{12}/I_o	C^{13}/I_o	N^{14}/I_o	N^{15}/I_o	O^{16}/I_o	O^{17}/I_o
0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
1.17×10^{-1}	4.99×10^{-6}	9.01×10^{-1}	8.05×10^{-2}	1.84×10^{-2}	6.88×10^{-7}	0.00	0.00
2.67×10^{-1}	4.42×10^{-5}	8.02×10^{-1}	1.29×10^{-1}	6.88×10^{-2}	2.70×10^{-6}	0.00	0.00
4.42×10^{-1}	1.60×10^{-4}	7.03×10^{-1}	1.52×10^{-1}	1.45×10^{-1}	5.77×10^{-6}	0.00	0.00
6.38×10^{-1}	4.04×10^{-4}	6.04×10^{-1}	1.56×10^{-1}	2.40×10^{-1}	9.65×10^{-6}	1.70×10^{-7}	0.00
8.48×10^{-1}	8.51×10^{-4}	5.05×10^{-1}	1.46×10^{-1}	3.50×10^{-1}	1.41×10^{-5}	3.50×10^{-7}	0.00
1.07	1.62×10^{-3}	4.05×10^{-1}	1.25×10^{-1}	4.69×10^{-1}	1.90×10^{-5}	6.70×10^{-7}	0.00
1.30	2.90×10^{-3}	3.06×10^{-1}	9.82×10^{-2}	5.96×10^{-1}	2.42×10^{-5}	1.21×10^{-6}	0.00
1.54	5.16×10^{-3}	2.07×10^{-1}	6.75×10^{-2}	7.26×10^{-1}	2.95×10^{-5}	2.14×10^{-6}	0.00
1.79	9.80×10^{-3}	1.08×10^{-1}	3.50×10^{-2}	8.57×10^{-1}	3.49×10^{-5}	4.07×10^{-6}	0.00
2.03	2.30×10^{-2}	2.66×10^{-2}	8.12×10^{-3}	9.65×10^{-1}	3.93×10^{-5}	9.54×10^{-6}	0.00
2.13	3.94×10^{-2}	1.09×10^{-2}	2.90×10^{-3}	9.86×10^{-1}	4.02×10^{-5}	1.64×10^{-5}	0.00
2.20	5.60×10^{-2}	8.73×10^{-3}	2.20×10^{-3}	9.89×10^{-1}	4.03×10^{-5}	2.32×10^{-5}	0.00
2.27	7.26×10^{-2}	8.44×10^{-3}	2.10×10^{-3}	9.89×10^{-1}	4.03×10^{-5}	3.01×10^{-5}	0.00
2.40	1.06×10^{-1}	8.39×10^{-3}	2.09×10^{-3}	9.89×10^{-1}	4.03×10^{-5}	4.39×10^{-5}	0.00
2.54	1.39×10^{-1}					5.77×10^{-5}	0.00
2.67	1.72×10^{-1}	CN "equilibrium"				7.14×10^{-5}	0.00
2.80	2.05×10^{-1}					8.52×10^{-5}	1.30×10^{-7}
2.94	2.39×10^{-1}	see text for discussion of this				9.89×10^{-5}	1.80×10^{-7}
3.07	2.72×10^{-1}					1.13×10^{-4}	2.30×10^{-7}
3.20	3.05×10^{-1}	region				1.26×10^{-4}	2.90×10^{-7}
3.60	4.04×10^{-1}					1.67×10^{-4}	5.00×10^{-7}
4.26	5.70×10^{-1}					2.36×10^{-4}	9.70×10^{-7}
4.93	7.36×10^{-1}					3.04×10^{-4}	1.58×10^{-6}

TABLE 3A (cont.)

RELATIVE ABUNDANCES AS FUNCTIONS OF PROTONS CONSUMED
PER INITIAL NUCLEUS FOR $T_6 = 20$, $C^{12}(0) = I_o$

$(p_o-p)/I_o$	He^4/I_o	C^{12}/I_o	C^{13}/I_o	N^{14}/I_o	N^{15}/I_o	O^{16}/I_o	O^{17}/I_o
8.58	1.65	8.39×10^{-3}	2.09×10^{-3}	9.89×10^{-1}	4.03×10^{-5}	6.76×10^{-4}	6.82×10^{-6}
1.52×10^{1}	3.31	8.38×10^{-3}	2.08×10^{-3}	9.88×10^{-1}	4.03×10^{-5}	1.34×10^{-3}	2.15×10^{-5}
2.18×10^{1}	4.96	8.38×10^{-3}	2.08×10^{-3}	9.87×10^{-1}	4.02×10^{-5}	1.98×10^{-3}	3.93×10^{-5}
2.85×10^{1}	6.62	8.37×10^{-3}	2.08×10^{-3}	9.87×10^{-1}	4.02×10^{-5}	2.61×10^{-3}	5.80×10^{-5}
3.51×10^{1}	8.27	8.37×10^{-3}	2.08×10^{-3}	9.86×10^{-1}	4.02×10^{-5}	3.22×10^{-3}	7.67×10^{-5}
1.01×10^{2}	2.48×10^{1}	8.32×10^{-3}	2.07×10^{-3}	9.81×10^{-1}	4.00×10^{-5}	8.49×10^{-3}	2.43×10^{-4}
1.67×10^{2}	4.12×10^{1}	8.29×10^{-3}	2.06×10^{-3}	9.77×10^{-1}	3.98×10^{-5}	1.25×10^{-2}	3.70×10^{-4}
2.32×10^{2}	5.76×10^{1}	8.26×10^{-3}	2.05×10^{-3}	9.74×10^{-1}	3.97×10^{-5}	1.56×10^{-2}	4.66×10^{-4}
2.97×10^{2}	7.39×10^{1}	8.24×10^{-3}	2.05×10^{-3}	9.71×10^{-1}	3.96×10^{-5}	1.79×10^{-2}	5.40×10^{-4}
3.95×10^{2}	9.83×10^{1}	8.22×10^{-3}	2.04×10^{-3}	9.69×10^{-1}	3.95×10^{-5}	2.04×10^{-2}	6.19×10^{-4}
5.25×10^{2}	1.31×10^{2}	8.20×10^{-3}	2.04×10^{-3}	9.67×10^{-1}	3.94×10^{-5}	2.25×10^{-2}	6.85×10^{-4}
6.55×10^{2}	1.63×10^{2}	8.19×10^{-3}	2.04×10^{-3}	9.65×10^{-1}	3.93×10^{-5}	2.37×10^{-2}	7.23×10^{-4}
1.30×10^{3}	3.25×10^{2}	8.17×10^{-3}	2.03×10^{-3}	9.64×10^{-1}	3.93×10^{-5}	2.53×10^{-2}	7.73×10^{-4}
∞	∞^*	8.17×10^{-3}	2.03×10^{-3}	9.64×10^{-1}	3.93×10^{-5}	2.54×10^{-2}	7.76×10^{-4}

*As $(p_o-p)/I_o$ approaches ∞, He^4/I_o approaches $\frac{1}{4}((p_o-p)/I_o - 2)$.

TABLE 3B

RATIOS OF ABUNDANCES AND TIME SCALES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $C^{12}(0) = I_o$

$(p_o-p)/I_o$	C^{13}/C^{12}	O^{17}/O^{16}	N/C	C/O	N/O	$t(\rho x_H)_o/100$ yr	t/τ^o_{12}
0.00	0.000	0.00	0.00			0.00	0.00
1.17×10^{-1}	0.089		1.87×10^{-2}			6.92×10^{2}	1.05×10^{-1}
2.67×10^{-1}	0.161		7.39×10^{-2}			1.47×10^{3}	2.22×10^{-1}
4.42×10^{-1}	0.217		1.69×10^{-1}			2.34×10^{3}	3.55×10^{-1}
6.38×10^{-1}	0.259		3.16×10^{-1}	4.47×10^{6}	1.41×10^{6}	3.36×10^{3}	5.08×10^{-1}
8.48×10^{-1}	0.289		5.37×10^{-1}	1.86×10^{6}	9.99×10^{5}	4.56×10^{3}	6.89×10^{-1}
1.07	0.309		8.85×10^{-1}	7.92×10^{5}	7.01×10^{5}	6.02×10^{3}	9.11×10^{-1}
1.30	0.321		1.47	3.34×10^{5}	4.92×10^{5}	7.91×10^{3}	1.20
1.54	0.326	4.02×10^{-5}	2.64	1.28×10^{5}	3.39×10^{5}	1.06×10^{4}	1.60
1.79	0.325	8.70×10^{-5}	6.01	3.51×10^{4}	2.11×10^{5}	1.51×10^{4}	2.29
2.03	0.306	1.96×10^{-4}	2.78×10^{1}	3.64×10^{3}	1.01×10^{5}	2.63×10^{4}	3.98
2.13	0.268	3.21×10^{-4}	7.17×10^{1}	8.39×10^{2}	6.01×10^{4}	3.94×10^{4}	5.97
2.20	0.252	4.49×10^{-4}	9.05×10^{1}	4.71×10^{2}	4.26×10^{4}	5.26×10^{4}	7.95
2.27	0.249	5.76×10^{-4}	9.39×10^{1}	3.50×10^{2}	3.29×10^{4}	6.57×10^{4}	9.94
2.40	0.249	8.30×10^{-4}	9.44×10^{1}	2.39×10^{2}	2.25×10^{4}	9.20×10^{4}	1.39×10^{1}
2.54		1.08×10^{-3}		1.82×10^{2}	1.71×10^{4}	1.18×10^{5}	1.79×10^{1}
2.67		1.33×10^{-3}		1.47×10^{2}	1.39×10^{4}	1.45×10^{5}	2.19×10^{1}
2.80	equilibrium	1.58×10^{-3}	equilibrium	1.23×10^{2}	1.16×10^{4}	1.71×10^{5}	2.59×10^{1}
2.94		1.83×10^{-3}		1.06×10^{2}	9.98×10^{3}	1.97×10^{5}	2.98×10^{1}
3.07		2.07×10^{-3}		9.28×10^{1}	8.77×10^{3}	2.23×10^{5}	3.38×10^{1}
3.20		2.31×10^{-3}		8.27×10^{1}	7.81×10^{3}	2.50×10^{5}	3.78×10^{1}
3.60		3.01×10^{-3}		6.24×10^{1}	5.89×10^{3}	3.29×10^{5}	4.97×10^{1}
4.26		4.13×10^{-3}		4.42×10^{1}	4.18×10^{3}	4.60×10^{5}	6.96×10^{1}
4.93		5.19×10^{-3}		3.43×10^{1}	3.24×10^{3}	5.91×10^{5}	8.95×10^{1}

TABLE 3B (cont.)

RATIOS OF ABUNDANCES AND TIME SCALES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $C^{12}(0) = I_o$

$(p_o - p)/I_o$	C^{13}/C^{12}	O^{17}/O^{16}	N/C	C/O	N/O	$t(\rho x_H)_o/100$ yr	t/τ^o_{12}
8.58		1.01×10^{-2}		1.53×10^{1}	1.45×10^{3}	1.31×10^{6}	1.99×10^{2}
1.52×10^{1}		1.61×10^{-2}		7.70	7.27×10^{2}	2.63×10^{6}	3.98×10^{2}
2.18×10^{1}		1.98×10^{-2}		5.17	4.88×10^{2}	3.94×10^{6}	5.97×10^{2}
2.85×10^{1}		2.22×10^{-2}		3.92	3.70×10^{2}	5.26×10^{6}	7.95×10^{2}
3.51×10^{1}		2.38×10^{-2}		3.17	2.99×10^{2}	6.57×10^{6}	9.94×10^{2}
1.01×10^{2}	equilibrium	2.86×10^{-2}	equilibrium	1.22	1.15×10^{2}	1.97×10^{7}	2.98×10^{3}
1.67×10^{2}		2.95×10^{-2}		8.03×10^{-1}	7.58×10^{1}	3.29×10^{7}	4.97×10^{3}
2.32×10^{2}		2.99×10^{-2}		6.43×10^{-1}	6.07×10^{1}	4.60×10^{7}	6.96×10^{3}
2.97×10^{2}		3.01×10^{-2}		5.57×10^{-1}	5.26×10^{1}	5.91×10^{7}	8.95×10^{3}
3.95×10^{2}		3.03×10^{-2}		4.88×10^{-1}	4.60×10^{1}	7.89×10^{7}	1.19×10^{4}
5.25×10^{2}		3.04×10^{-2}		4.41×10^{-1}	4.17×10^{1}	1.05×10^{8}	1.59×10^{4}
6.55×10^{2}		3.05×10^{-2}		4.18×10^{-1}	3.95×10^{1}	1.31×10^{8}	1.99×10^{4}
1.30×10^{3}	0.249	3.05×10^{-2}	9.44×10^{1}	3.91×10^{-1}	3.70×10^{1}	2.63×10^{8}	3.98×10^{4}
∞	0.249	3.05×10^{-2}	9.44×10^{1}	3.90×10^{-1}	3.68×10^{1}	∞	∞

TABLE 4A

RELATIVE ABUNDANCES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $O^{16}(0) = I_o$

$(p_o-p)/I_o$	He^4/I_o	C^{12}/I_o	C^{13}/I_o	N^{14}/I_o	N^{15}/I_o	O^{16}/I_o	O^{17}/I_o
0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
7.05×10^{-5}	0.00	0.00	0.00	1.30×10^{-7}	0.00	1.00	9.14×10^{-5}
2.98×10^{-4}	0.00	0.00	0.00	1.49×10^{-6}	0.00	1.00	3.03×10^{-4}
5.34×10^{-4}	4.81×10^{-6}	0.00	0.00	4.51×10^{-6}	0.00	9.99×10^{-1}	5.24×10^{-4}
8.02×10^{-4}	1.01×10^{-5}	0.00	0.00	1.01×10^{-5}	0.00	9.99×10^{-1}	7.82×10^{-4}
1.05×10^{-3}	1.32×10^{-5}	1.10×10^{-7}	0.00	1.79×10^{-5}	0.00	9.99×10^{-1}	1.04×10^{-3}
1.34×10^{-3}	2.63×10^{-5}	1.90×10^{-7}	0.00	2.79×10^{-5}	1.10×10^{-9}	9.99×10^{-1}	1.29×10^{-3}
1.89×10^{-3}	5.25×10^{-5}	4.00×10^{-7}	0.00	5.43×10^{-5}	2.20×10^{-9}	9.98×10^{-1}	1.79×10^{-3}
2.48×10^{-3}	9.47×10^{-5}	6.80×10^{-7}	1.60×10^{-7}	8.93×10^{-5}	3.60×10^{-9}	9.98×10^{-1}	2.29×10^{-3}
3.06×10^{-3}	1.41×10^{-4}	1.03×10^{-6}	2.50×10^{-7}	1.33×10^{-4}	5.40×10^{-9}	9.97×10^{-1}	2.77×10^{-3}
3.66×10^{-3}	1.97×10^{-4}	1.45×10^{-6}	3.50×10^{-7}	1.84×10^{-4}	7.50×10^{-9}	9.97×10^{-1}	3.24×10^{-3}
4.27×10^{-3}	2.64×10^{-4}	1.93×10^{-6}	4.70×10^{-7}	2.44×10^{-4}	9.90×10^{-9}	9.96×10^{-1}	3.71×10^{-3}
4.90×10^{-3}	3.42×10^{-4}	2.50×10^{-6}	6.10×10^{-7}	3.11×10^{-4}	1.27×10^{-8}	9.96×10^{-1}	4.17×10^{-3}
5.55×10^{-3}	4.30×10^{-4}	3.10×10^{-6}	7.60×10^{-7}	3.86×10^{-4}	1.57×10^{-8}	9.95×10^{-1}	4.62×10^{-3}
7.59×10^{-3}	7.53×10^{-4}	5.40×10^{-6}	1.32×10^{-6}	6.57×10^{-4}	2.67×10^{-8}	9.93×10^{-1}	5.92×10^{-3}
1.14×10^{-2}	1.51×10^{-3}	1.03×10^{-5}	2.55×10^{-6}	1.25×10^{-3}	5.10×10^{-8}	9.91×10^{-1}	7.94×10^{-3}
1.59×10^{-2}	2.55×10^{-3}	1.67×10^{-5}	4.13×10^{-6}	2.01×10^{-3}	8.19×10^{-8}	9.88×10^{-1}	9.79×10^{-3}
5.51×10^{-2}	1.38×10^{-2}	7.20×10^{-5}	1.79×10^{-5}	8.56×10^{-3}	3.49×10^{-7}	9.74×10^{-1}	1.74×10^{-2}
2.14×10^{-1}	6.12×10^{-2}	2.27×10^{-4}	5.64×10^{-5}	2.69×10^{-2}	1.09×10^{-6}	9.49×10^{-1}	2.43×10^{-2}
5.14×10^{-1}	1.47×10^{-1}	4.14×10^{-4}	1.03×10^{-4}	4.89×10^{-2}	1.99×10^{-6}	9.24×10^{-1}	2.68×10^{-2}
9.67×10^{-1}	2.72×10^{-1}	6.10×10^{-4}	1.52×10^{-4}	7.21×10^{-2}	2.94×10^{-6}	9.00×10^{-1}	2.74×10^{-2}
1.58	4.36×10^{-1}	8.08×10^{-4}	2.01×10^{-4}	9.54×10^{-2}	3.89×10^{-6}	8.76×10^{-1}	2.72×10^{-2}
1.56×10^{1}	4.05	2.56×10^{-3}	6.35×10^{-4}	3.01×10^{-1}	1.23×10^{-5}	6.74×10^{-1}	2.12×10^{-2}
4.17×10^{1}	1.06×10^{1}	3.89×10^{-3}	9.67×10^{-4}	4.59×10^{-1}	1.87×10^{-5}	5.20×10^{-1}	1.64×10^{-2}

TABLE 4A (cont.)

RELATIVE ABUNDANCES AS FUNCTIONS OF PROTONS CONSUMED
PER INITIAL NUCLEUS FOR $T_6 = 20$, $O^{16}(0) = I_o$

$(p_o-p)/I_o$	He^4/I_o	C^{12}/I_o	C^{13}/I_o	N^{14}/I_o	N^{15}/I_o	O^{16}/I_o	O^{17}/I_o
7.69×10^{1}	1.95×10^{1}	4.91×10^{-3}	1.22×10^{-3}	5.79×10^{-1}	2.36×10^{-5}	4.03×10^{-1}	1.27×10^{-2}
1.19×10^{2}	3.01×10^{1}	5.68×10^{-3}	1.41×10^{-3}	6.70×10^{-1}	2.73×10^{-5}	3.13×10^{-1}	9.84×10^{-3}
1.92×10^{2}	4.84×10^{1}	6.52×10^{-3}	1.62×10^{-3}	7.68×10^{-1}	3.13×10^{-5}	2.17×10^{-1}	6.81×10^{-3}
3.01×10^{2}	7.58×10^{1}	7.21×10^{-3}	1.79×10^{-3}	8.50×10^{-1}	3.46×10^{-5}	1.37×10^{-1}	4.29×10^{-3}
4.19×10^{2}	1.05×10^{2}	7.61×10^{-3}	1.89×10^{-3}	8.98×10^{-1}	3.66×10^{-5}	9.01×10^{-2}	2.82×10^{-3}
1.05×10^{3}	2.63×10^{2}	8.14×10^{-3}	2.02×10^{-3}	9.59×10^{-1}	3.91×10^{-5}	2.97×10^{-2}	9.12×10^{-4}
∞	∞	8.17×10^{-3}	2.03×10^{-3}	9.64×10^{-1}	3.93×10^{-5}	2.54×10^{-2}	7.76×10^{-4}

TABLE 4B

RATIOS OF ABUNDANCES AND TIME SCALES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $O^{16}(0) = I_o$

$(p_o - p)/I_o$	C^{13}/C^{12}	O^{17}/O^{16}	N/C	C/O	N/O	$t(\rho x_H)_o/100$ yr	t/τ^o_{16}
0.00		0.00		0.00	0.00	0.00	0.00
7.05×10^{-5}		9.14×10^{-5}	2.42×10^{2}	0.00	1.30×10^{-7}	4.56×10^{3}	9.30×10^{-5}
2.98×10^{-4}	0.213	3.03×10^{-4}	2.08×10^{2}	0.00	1.49×10^{-6}	1.51×10^{4}	3.09×10^{-4}
5.34×10^{-4}	0.219	5.24×10^{-4}	1.55×10^{2}	0.00	4.51×10^{-6}	2.63×10^{4}	5.37×10^{-4}
8.02×10^{-4}	0.227	7.83×10^{-4}	1.34×10^{2}	0.00	1.01×10^{-5}	3.94×10^{4}	8.05×10^{-4}
1.05×10^{-3}	0.232	1.04×10^{-3}	1.23×10^{2}	1.10×10^{-7}	1.79×10^{-5}	5.26×10^{4}	1.07×10^{-3}
1.34×10^{-3}	0.236	1.29×10^{-3}	1.17×10^{2}	1.90×10^{-7}	2.79×10^{-5}	6.57×10^{4}	1.34×10^{-3}
1.89×10^{-3}	0.240	1.80×10^{-3}	1.10×10^{2}	4.00×10^{-7}	5.43×10^{-5}	9.20×10^{4}	1.88×10^{-3}
2.48×10^{-3}	0.242	2.29×10^{-3}	1.06×10^{2}	8.40×10^{-7}	8.93×10^{-5}	1.18×10^{5}	2.41×10^{-3}
3.06×10^{-3}	0.243	2.78×10^{-3}	1.04×10^{2}	1.28×10^{-6}	1.33×10^{-4}	1.45×10^{5}	2.95×10^{-3}
3.66×10^{-3}	0.244	3.25×10^{-3}	1.02×10^{2}	1.80×10^{-6}	1.84×10^{-4}	1.71×10^{5}	3.49×10^{-3}
4.27×10^{-3}	0.244	3.72×10^{-3}	1.01×10^{2}	2.40×10^{-6}	2.44×10^{-4}	1.97×10^{5}	4.02×10^{-3}
4.90×10^{-3}	0.245	4.19×10^{-3}	1.00×10^{2}	3.11×10^{-6}	3.11×10^{-4}	2.23×10^{5}	4.56×10^{-3}
5.55×10^{-3}	0.245	4.64×10^{-3}	9.97×10^{1}	3.86×10^{-6}	3.86×10^{-4}	2.50×10^{5}	5.10×10^{-3}
7.59×10^{-3}	0.246	5.96×10^{-3}	9.84×10^{1}	6.72×10^{-6}	6.57×10^{-4}	3.29×10^{5}	6.71×10^{-3}
1.14×10^{-2}	0.247	8.01×10^{-3}	9.72×10^{1}	1.29×10^{-5}	1.25×10^{-3}	4.60×10^{5}	9.39×10^{-3}
1.59×10^{-2}	0.247	9.90×10^{-3}	9.65×10^{1}	2.09×10^{-5}	2.02×10^{-3}	5.91×10^{5}	1.21×10^{-2}
5.51×10^{-2}	0.248	1.79×10^{-2}	9.53×10^{1}	9.07×10^{-5}	8.64×10^{-3}	1.31×10^{6}	2.68×10^{-2}
2.14×10^{-1}	0.248	2.56×10^{-2}	9.48×10^{1}	2.91×10^{-4}	2.76×10^{-2}	2.63×10^{6}	5.37×10^{-2}
5.14×10^{-1}	0.248	2.90×10^{-2}	9.47×10^{1}	5.44×10^{-4}	5.14×10^{-2}	3.94×10^{6}	8.05×10^{-2}
9.67×10^{-1}	0.249	3.04×10^{-2}	9.46×10^{1}	8.22×10^{-4}	7.77×10^{-2}	5.26×10^{6}	1.07×10^{-1}
1.58	0.249	3.10×10^{-2}	9.45×10^{1}	1.12×10^{-3}	1.06×10^{-1}	6.57×10^{6}	1.34×10^{-1}
1.56×10^{1}	0.249	3.15×10^{-2}	9.45×10^{1}	4.59×10^{-3}	4.33×10^{-1}	1.97×10^{7}	4.02×10^{-1}
4.17×10^{1}	0.249	3.15×10^{-2}	9.44×10^{1}	9.05×10^{-3}	8.55×10^{-1}	3.29×10^{7}	6.71×10^{-1}

TABLE 4B (cont.)

RATIOS OF ABUNDANCES AND TIME SCALES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $O^{16}(0) = I_o$

$(p_o-p)/I_o$	C^{13}/C^{12}	O^{17}/O^{16}	N/C	C/O	N/O	$t(px_H)_o/100$ yr	t/τ^o_{16}
7.69×10^{1}	0.249	3.15×10^{-2}	9.44×10^{1}	1.48×10^{-2}	1.39	4.60×10^{7}	9.39×10^{-1}
1.19×10^{2}	0.249	3.15×10^{-2}	9.44×10^{1}	2.20×10^{-2}	2.08	5.91×10^{7}	1.21
1.92×10^{2}	0.249	3.14×10^{-2}	9.44×10^{1}	3.64×10^{-2}	3.44	7.89×10^{7}	1.61
3.01×10^{2}	0.249	3.14×10^{-2}	9.44×10^{1}	6.39×10^{-2}	6.03	1.05×10^{8}	2.15
4.19×10^{2}	0.249	3.13×10^{-2}	9.44×10^{1}	1.02×10^{-1}	9.66	1.31×10^{8}	2.68
1.05×10^{3}	0.249	3.07×10^{-2}	9.44×10^{1}	3.32×10^{-1}	3.13×10^{1}	2.63×10^{8}	5.37
∞	0.249	3.05×10^{-2}	9.44×10^{1}	3.90×10^{-1}	3.68×10^{1}	∞	∞

TABLE 5A

RELATIVE ABUNDANCES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $C^{12}(0) = O^{16}(0) = 0.5\ I_o$

$(p_o-p)/I_o$	He^4/I_o	C^{12}/I_o	C^{13}/I_o	N^{14}/I_o	N^{15}/I_o	O^{16}/I_o	O^{17}/I_o
0.00	0.00	5.00×10^{-1}	0.00	0.00	0.00	5.00×10^{-1}	0.00
5.87×10^{-2}	2.00×10^{-6}	4.51×10^{-1}	4.03×10^{-2}	9.20×10^{-3}	3.44×10^{-7}	5.00×10^{-1}	7.00×10^{-6}
1.33×10^{-1}	2.10×10^{-5}	4.01×10^{-1}	6.46×10^{-2}	3.44×10^{-2}	1.35×10^{-6}	5.00×10^{-1}	1.50×10^{-5}
2.21×10^{-1}	7.90×10^{-5}	3.51×10^{-1}	7.62×10^{-2}	7.23×10^{-2}	2.89×10^{-6}	5.00×10^{-1}	2.40×10^{-5}
3.19×10^{-1}	2.02×10^{-4}	3.02×10^{-1}	7.82×10^{-2}	1.20×10^{-1}	4.82×10^{-6}	5.00×10^{-1}	3.40×10^{-5}
4.24×10^{-1}	4.23×10^{-4}	2.52×10^{-1}	7.29×10^{-2}	1.75×10^{-1}	7.06×10^{-6}	5.00×10^{-1}	4.60×10^{-5}
5.35×10^{-1}	8.07×10^{-4}	2.03×10^{-1}	6.26×10^{-2}	2.35×10^{-1}	9.51×10^{-6}	5.00×10^{-1}	6.00×10^{-5}
6.51×10^{-1}	1.45×10^{-3}	1.53×10^{-1}	4.91×10^{-2}	2.98×10^{-1}	1.21×10^{-5}	5.00×10^{-1}	7.90×10^{-5}
7.70×10^{-1}	2.58×10^{-3}	1.03×10^{-1}	3.37×10^{-2}	3.63×10^{-1}	1.48×10^{-5}	5.00×10^{-1}	1.06×10^{-4}
8.95×10^{-1}	4.90×10^{-3}	5.38×10^{-2}	1.75×10^{-2}	4.29×10^{-1}	1.74×10^{-5}	5.00×10^{-1}	1.51×10^{-4}
1.02	1.15×10^{-2}	1.33×10^{-2}	4.06×10^{-3}	4.83×10^{-1}	1.97×10^{-5}	5.00×10^{-1}	2.62×10^{-4}
1.07	1.97×10^{-2}	5.43×10^{-3}	1.45×10^{-3}	4.93×10^{-1}	2.01×10^{-5}	5.00×10^{-1}	3.91×10^{-4}
1.10	2.80×10^{-2}	4.36×10^{-3}	1.10×10^{-3}	4.95×10^{-1}	2.01×10^{-5}	4.99×10^{-1}	5.19×10^{-4}
1.14	3.63×10^{-2}	4.22×10^{-3}	1.05×10^{-3}	4.95×10^{-1}	2.02×10^{-5}	4.99×10^{-1}	6.46×10^{-4}
1.20	5.29×10^{-2}	4.20×10^{-3}	1.04×10^{-3}	4.95×10^{-1}	2.02×10^{-5}	4.99×10^{-1}	8.96×10^{-4}
1.27	6.95×10^{-2}	4.20×10^{-3}	1.04×10^{-3}	4.95×10^{-1}	2.02×10^{-5}	4.99×10^{-1}	1.14×10^{-3}
1.34	8.61×10^{-2}	4.20×10^{-3}	1.04×10^{-3}	4.95×10^{-1}	2.02×10^{-5}	4.99×10^{-1}	1.38×10^{-3}
1.40	1.03×10^{-1}					4.98×10^{-1}	1.62×10^{-3}
1.47	1.19×10^{-1}	CN "equilibrium" see				4.98×10^{-1}	1.85×10^{-3}
1.54	1.36×10^{-1}					4.98×10^{-1}	2.08×10^{-3}
1.60	1.53×10^{-1}	text for discussion				4.98×10^{-1}	2.31×10^{-3}
1.80	2.03×10^{-1}					4.97×10^{-1}	2.96×10^{-3}
2.14	2.86×10^{-1}		1.04×10^{-3}			4.96×10^{-1}	3.97×10^{-3}
2.47	3.69×10^{-1}	4.20×10^{-3}	1.05×10^{-3}	4.96×10^{-1}	2.02×10^{-5}	4.94×10^{-1}	4.89×10^{-3}

TABLE 5A (cont.)

RELATIVE ABUNDANCES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $C^{12}(0) = O^{16}(0) = 0.5\ I_o$

$(p_o-p)/I_o$	He^4/I_o	C^{12}/I_o	C^{13}/I_o	N^{14}/I_o	N^{15}/I_o	O^{16}/I_o	O^{17}/I_o
4.32	8.31×10^{-1}	4.23×10^{-3}	1.05×10^{-3}	4.99×10^{-1}	2.03×10^{-5}	4.87×10^{-1}	8.71×10^{-3}
7.71	1.68	4.30×10^{-3}	1.07×10^{-3}	5.07×10^{-1}	2.07×10^{-5}	4.75×10^{-1}	1.22×10^{-2}
1.12×10^{1}	2.56	4.40×10^{-3}	1.09×10^{-3}	5.18×10^{-1}	2.11×10^{-5}	4.63×10^{-1}	1.34×10^{-2}
1.47×10^{1}	3.45	4.49×10^{-3}	1.12×10^{-3}	5.29×10^{-1}	2.16×10^{-5}	4.51×10^{-1}	1.37×10^{-2}
1.83×10^{1}	4.36	4.59×10^{-3}	1.14×10^{-3}	5.41×10^{-1}	2.20×10^{-5}	4.40×10^{-1}	1.36×10^{-2}
5.83×10^{1}	1.44×10^{1}	5.44×10^{-3}	1.35×10^{-3}	6.41×10^{-1}	2.61×10^{-5}	3.41×10^{-1}	1.07×10^{-2}
1.04×10^{2}	2.59×10^{1}	6.09×10^{-3}	1.51×10^{-3}	7.18×10^{-1}	2.92×10^{-5}	2.66×10^{-1}	8.37×10^{-3}
1.55×10^{2}	3.85×10^{1}	6.58×10^{-3}	1.64×10^{-3}	7.76×10^{-1}	3.16×10^{-5}	2.09×10^{-1}	6.57×10^{-3}
2.08×10^{2}	5.20×10^{1}	6.96×10^{-3}	1.73×10^{-3}	8.21×10^{-1}	3.34×10^{-5}	1.65×10^{-1}	5.19×10^{-3}
2.94×10^{2}	7.33×10^{1}	7.37×10^{-3}	1.83×10^{-3}	8.68×10^{-1}	3.54×10^{-5}	1.19×10^{-1}	3.72×10^{-3}
4.13×10^{2}	1.03×10^{2}	7.70×10^{-3}	1.92×10^{-3}	9.08×10^{-1}	3.70×10^{-5}	7.96×10^{-2}	2.49×10^{-3}
5.37×10^{2}	1.34×10^{2}	7.90×10^{-3}	1.96×10^{-3}	9.31×10^{-1}	3.79×10^{-5}	5.69×10^{-2}	1.77×10^{-3}
1.18×10^{3}	2.94×10^{2}	8.16×10^{-3}	2.03×10^{-3}	9.61×10^{-1}	3.92×10^{-5}	2.75×10^{-2}	8.42×10^{-4}
∞	∞	8.17×10^{-3}	2.03×10^{-3}	9.64×10^{-1}	3.93×10^{-5}	2.54×10^{-2}	7.76×10^{-4}

TABLE 5B

RATIOS OF ABUNDANCES AND TIME SCALES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $C^{12}(0) = O^{16}(0) = 0.5\ I_o$

$(p_o-p)/I_o$	C^{13}/C^{12}	O^{17}/O^{16}	N/C	C/O	N/O	$t(\rho x_H)_o/100$ yr	t/τ^o_{12}
0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
5.87×10^{-2}	0.089	1.40×10^{-5}	1.87×10^{-2}	9.82×10^{-1}	1.84×10^{-2}	6.92×10^{2}	1.05×10^{-1}
1.33×10^{-1}	0.161	2.90×10^{-5}	7.39×10^{-2}	9.31×10^{-1}	6.88×10^{-2}	1.47×10^{3}	2.22×10^{-1}
2.21×10^{-1}	0.217	4.70×10^{-5}	1.69×10^{-1}	8.55×10^{-1}	1.45×10^{-1}	2.34×10^{3}	3.55×10^{-1}
3.19×10^{-1}	0.259	6.70×10^{-5}	3.16×10^{-1}	7.60×10^{-1}	2.40×10^{-1}	3.36×10^{3}	5.08×10^{-1}
4.24×10^{-1}	0.289	9.10×10^{-5}	5.37×10^{-1}	6.50×10^{-1}	3.50×10^{-1}	4.56×10^{3}	6.89×10^{-1}
5.35×10^{-1}	0.309	1.21×10^{-4}	8.85×10^{-1}	5.31×10^{-1}	4.69×10^{-1}	6.02×10^{3}	9.11×10^{-1}
6.51×10^{-1}	0.321	1.59×10^{-4}	1.47	4.04×10^{-1}	5.96×10^{-1}	7.91×10^{3}	1.20
7.70×10^{-1}	0.326	2.12×10^{-4}	2.64	2.74×10^{-1}	7.26×10^{-1}	1.06×10^{4}	1.60
8.95×10^{-1}	0.325	3.03×10^{-4}	6.01	1.43×10^{-1}	8.57×10^{-1}	1.51×10^{4}	2.29
1.02	0.306	5.24×10^{-4}	2.78×10^{1}	3.47×10^{-2}	9.65×10^{-1}	2.63×10^{4}	3.98
1.07	0.268	7.83×10^{-4}	7.17×10^{1}	1.38×10^{-2}	9.86×10^{-1}	3.94×10^{4}	5.97
1.10	0.252	1.04×10^{-3}	9.05×10^{1}	1.09×10^{-2}	9.89×10^{-1}	5.26×10^{4}	7.95
1.14	0.249	1.29×10^{-3}	9.39×10^{1}	1.05×10^{-2}	9.89×10^{-1}	6.57×10^{4}	9.94
1.20	0.249	1.80×10^{-3}	9.44×10^{1}	1.05×10^{-2}	9.90×10^{-1}	9.20×10^{4}	1.39×10^{1}
1.27	0.249	2.29×10^{-3}	9.44×10^{1}			1.18×10^{5}	1.79×10^{1}
1.34	0.249	2.78×10^{-3}	9.44×10^{1}	CN		1.45×10^{5}	2.19×10^{1}
1.40	equilibrium	3.25×10^{-3}	9.44×10^{1}			1.71×10^{5}	2.59×10^{1}
1.47		3.72×10^{-3}	9.44×10^{1}	"equilibrium"		1.97×10^{5}	2.98×10^{1}
1.54		4.19×10^{-3}				2.23×10^{5}	3.38×10^{1}
1.60		4.64×10^{-3}	see text for discussion			2.50×10^{5}	3.78×10^{1}
1.80		5.96×10^{-3}				3.29×10^{5}	4.97×10^{1}
2.14		8.01×10^{-3}			9.92×10^{-1}	4.60×10^{5}	6.96×10^{1}
2.47		9.90×10^{-3}			9.93×10^{-1}	5.91×10^{5}	8.95×10^{1}

TABLE 5B (cont.)

RATIOS OF ABUNDANCES AND TIME SCALES AS FUNCTIONS OF PROTONS CONSUMED PER INITIAL NUCLEUS FOR $T_6 = 20$, $C^{12}(0) = O^{16}(0) = 0.5\ I_o$

$(p_o-p)/I_o$	C^{13}/C^{12}	O^{17}/O^{16}	N/C	C/O	N/O	$t(\rho x_H)_o$/100 yr	t/τ^o_{12}
4.32		1.79×10^{-2}		1.06×10^{-2}	1.01	1.31×10^{6}	1.99×10^{2}
7.71		2.56×10^{-2}		1.10×10^{-2}	1.04	2.63×10^{6}	3.98×10^{2}
1.12×10^{1}		2.89×10^{-2}		1.15×10^{-2}	1.09	3.94×10^{6}	5.97×10^{2}
1.47×10^{1}	equilibrium	3.04×10^{-2}		1.21×10^{-2}	1.14	5.26×10^{6}	7.95×10^{2}
1.83×10^{1}		3.10×10^{-2}		1.26×10^{-2}	1.19	6.57×10^{6}	9.94×10^{2}
5.83×10^{1}		3.15×10^{-2}		1.93×10^{-2}	1.82	1.97×10^{7}	2.98×10^{3}
1.04×10^{2}		3.14×10^{-2}		2.77×10^{-2}	2.61	3.29×10^{7}	4.97×10^{3}
1.55×10^{2}		3.14×10^{-2}		3.81×10^{-2}	3.60	4.60×10^{7}	6.96×10^{3}
2.08×10^{2}		3.14×10^{-2}		5.09×10^{-2}	4.81	5.91×10^{7}	8.95×10^{3}
2.94×10^{2}		3.13×10^{-2}		7.52×10^{-2}	7.10	7.89×10^{7}	1.19×10^{4}
4.13×10^{2}		3.12×10^{-2}		1.17×10^{-1}	1.11×10^{1}	1.05×10^{8}	1.59×10^{4}
5.37×10^{2}		3.11×10^{-2}		1.68×10^{-1}	1.59×10^{1}	1.31×10^{8}	1.99×10^{4}
1.18×10^{3}	0.249	3.06×10^{-2}	9.44×10^{1}	3.59×10^{-1}	3.39×10^{1}	2.63×10^{8}	3.98×10^{4}
∞	0.249	3.05×10^{-2}	9.44×10^{1}	3.90×10^{-1}	3.68×10^{1}	∞	∞

TABLE 6A

LACK OF SENSITIVITY TO TEMPERATURE

T_6 =	10	17	20	30	50
$(p_o - p)/I_o$	0.578	0.581	0.583	0.582	0.583
C^{13}/C^{12}	0.252	0.249	0.249	0.248	0.248
N/C	0.263	0.267	0.270	0.271	0.268
t/τ^o_{12}	0.460	0.456	0.465	0.468	0.459
$(p_o - p)/I_o$	1.12	1.12	1.12	1.13	1.15
C^{13}/C^{12}	0.318	0.314	0.313	0.312	0.311
N/C	1.00	1.00	1.00	1.00	1.00
t/τ^o_{12}	0.966	0.957	0.971	0.982	0.974
$(p_o - p)/I_o$	1.71	1.68	1.65	1.58	1.53
C^{13}/C^{12}	0.335	0.328	0.327	0.323	0.320
N/C	4.84	4.04	3.47	2.78	2.25
t/τ^o_{12}	2.06	1.91	1.85	1.67	1.50
$(p_o - p)/I_o$	2.09	2.28	2.35	2.70	3.35
C^{13}/C^{12}	0.252	0.249	0.249	0.248	0.248
N/C	386	127	94.4	48.5	24.3
t/τ^o_{12}	~ 12	~ 12	~ 12	~ 12	~ 12

TABLE 6B

TIME SCALES IN APPROACH TO EQUILIBRIUM

T_6	$t_1(\rho x_H)_o/100$ (a)	t_1/τ^o_{12}	$t_2(\rho x_H)_o/100$ (b)	t_2/τ^o_{12}
20	3.08×10^{3}	0.466	6.52×10^{3}	0.976
50	8.58×10^{-3}	0.461	1.81×10^{-2}	0.974
T_6	C^{13}/C^{12} (max.)	$t_3(\rho x_H)_o/100$ (c)	t_3/τ^o_{12}	
20	0.327	1.19×10^{4}	1.80	
50	0.320	3.12×10^{-2}	1.68	
T_6	$t_4(\rho x_H)_o/100$ (d)	t_4/τ^o_{12}	$t_5(\rho x_H)_o/100$ (e)	t_5/τ^o_{16}
20	6.57×10^{4}	9.95	1.07×10^{6}	0.022
50	1.78×10^{-1}	9.57	1.89×10^{-1}	0.020

(a) t_1 = time in years at which $C^{13}/C^{12} = 0.25$ in rise to maximum.

(b) t_2 = time in years at which N/C = 1.

(c) t_3 = time in years at which C^{13}/C^{12} attains maximum value.

(d) t_4 = time in years at which CN "equilibrium" occurs.

(e) t_5 = time in years at which N/O = 1.

TABLE 7

C^{12} ABUNDANCES AFTER CONVECTION

	Seesaw Convection				Sinusoidal Convection			
T_o	20×10^6 °K	20×10^6 °K	50×10^6 °K	50×10^6 °K	20×10^6 °K	20×10^6 °K	50×10^6 °K	50×10^6 °K
r	14	18	14	18	14	18	14	18
t_f								
τ_{12}^{oo}	0.931	0.946	0.931	0.946	0.857	0.873	0.857	0.873
$5\tau_{12}^{oo}$	.700	.758	.701	.759	.462	.508	.467	.512
$6\tau_{12}^{oo}$	.652	.717	.654	.718	.390	.444	.402	.449
$7\tau_{12}^{oo}$	.607	.678	.610	.680	.340	.389	.348	.394
$8\tau_{12}^{oo}$	.566	.642	.568	.644	.293	.340	.301	.347
$10\tau_{12}^{oo}$	.491	.575	.495	.577	.216	.260	.227	.270
$15\tau_{12}^{oo}$	0.345	0.436	0.352	0.441	0.104	0.135	0.120	0.150

TABLE 8

CONTRIBUTIONS OF RESONANCE LEVELS

Reaction	Resonance E_p (lab) keV	$\omega\Gamma_\gamma$(lab) eV	Contribution $\log \tau\rho x_H$ yr	Reference
$C^{12}(p,\gamma)N^{13}$	456	0.67	$-12.53 + \frac{3}{2}\log T_9 + \frac{2.14}{T_9}$	A-S, L 1959*
$C^{12}(p,\gamma)N^{13}$	1698	1.39	$-12.85 + \frac{3}{2}\log T_9 + \frac{7.96}{T_9}$	A-S, L 1959*
$C^{13}(p,\gamma)N^{14}$	448	0.008	$-10.60 + \frac{3}{2}\log T_9 + \frac{2.11}{T_9}$	A-S, L 1959*
$C^{13}(p,\gamma)N^{14}$	554	7.5	$-13.58 + \frac{3}{2}\log T_9 + \frac{2.61}{T_9}$	Vogl 1963
$C^{13}(p,\gamma)N^{14}$	1250†	15.0†	$-13.88 + \frac{3}{2}\log T_9 + \frac{5.89}{T_9}$	A-S, L 1959*
$C^{13}(p,\gamma)N^{14}$	1747.3	14.8	$-13.87 + \frac{3}{2}\log T_9 + \frac{8.24}{T_9}$	A-S, L 1959*
$C^{13}(p,\gamma)N^{14}$	2112	6.2	$-13.49 + \frac{3}{2}\log T_9 + \frac{9.96}{T_9}$	A-S, L 1959*
$C^{13}(p,\gamma)N^{14}$	3110	17	$-13.93 + \frac{3}{2}\log T_9 + \frac{14.66}{T_9}$	A-S, L 1959*
$N^{14}(p,\gamma)O^{15}$	278	1.45×10^{-2}	$-10.86 + \frac{3}{2}\log T_9 + \frac{1.32}{T_9}$	H, B 1964‡
$N^{14}(p,\gamma)O^{15}$	1060	0.394	$-12.30 + \frac{3}{2}\log T_9 + \frac{5.02}{T_9}$	H, B 1964‡
$N^{14}(p,\gamma)O^{15}$	1800§	0.56	$-12.45 + \frac{3}{2}\log T_9 + \frac{8.53}{T_9}$	A-S, L 1959* corrected by H, B 1964‡
$N^{14}(p,\gamma)O^{15}$	2600††	32.2	$-14.21 + \frac{3}{2}\log T_9 + \frac{12.32}{T_9}$	A-S, L 1959* corrected by H, B 1964‡

* Ajzenberg-Selove, F. and Lauritsen, T. 1959.

† Four levels at 1160, 1250, 1466, and 1550 keV with $\omega\Gamma_\gamma$ values of 1.3, 12.8, 0.72, and 0.13 respectively combined at 1250 keV with value of sum of $\omega\Gamma_\gamma$ values.

‡ Hebbard, D. F. and Bailey, G. M. 1964.

§ Three levels at 1550, 1748, and 1815 keV with $\omega\Gamma_\gamma$ values of 0.16, 0.21, and 0.52 respectively combined after application of correction factor of 1/1.6 at 1800 keV with value of sum of corrected $\omega\Gamma_\gamma$ values.

†† Three levels at 2356, 2489, and 2600 keV with $\omega\Gamma_\gamma$ values of 2.4, 3.3, and 46 respectively combined after application of correction factor of 1/1.6 on $\omega\Gamma_\gamma$ values at 2600 keV with value of sum of corrected $\omega\Gamma_\gamma$ values.

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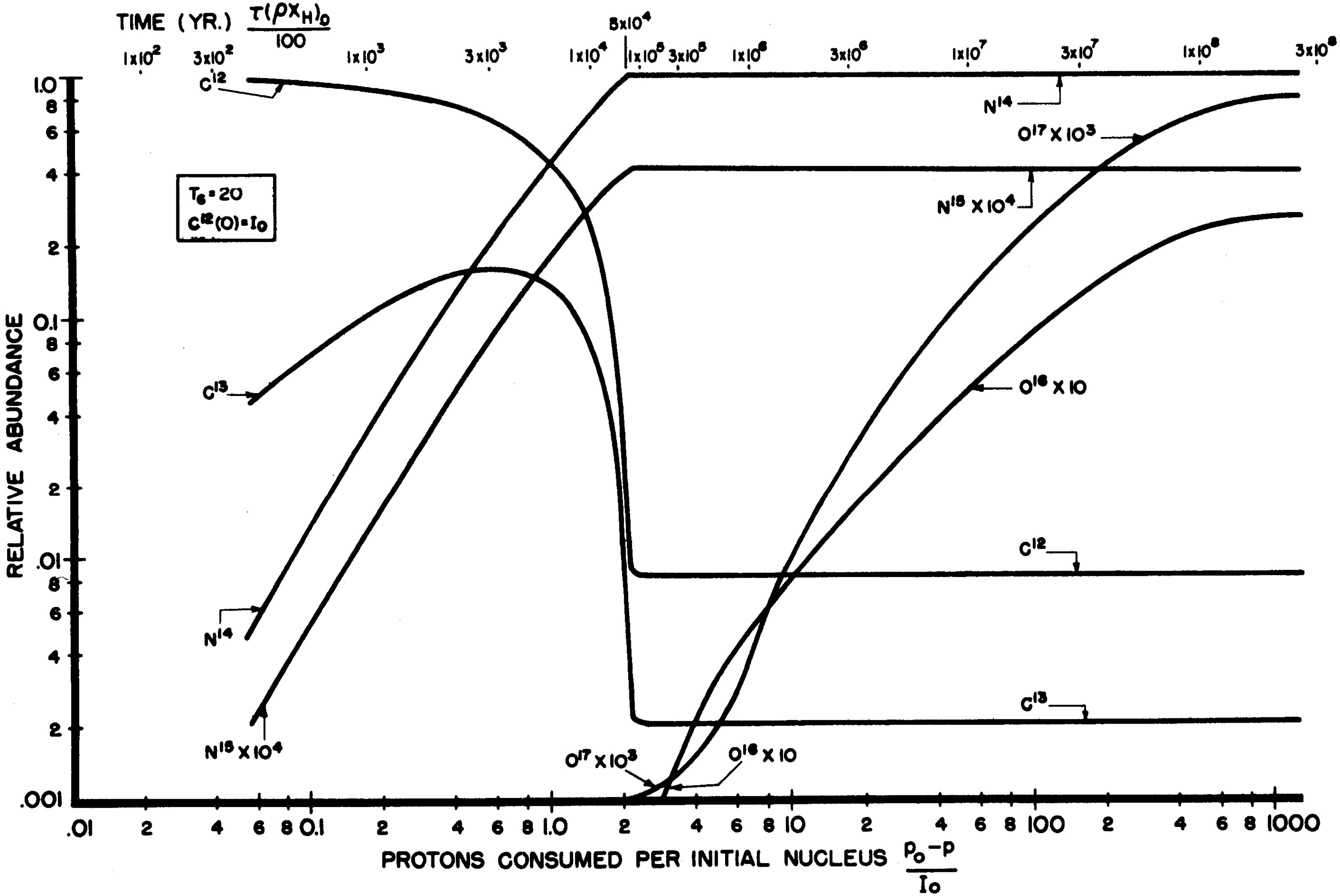
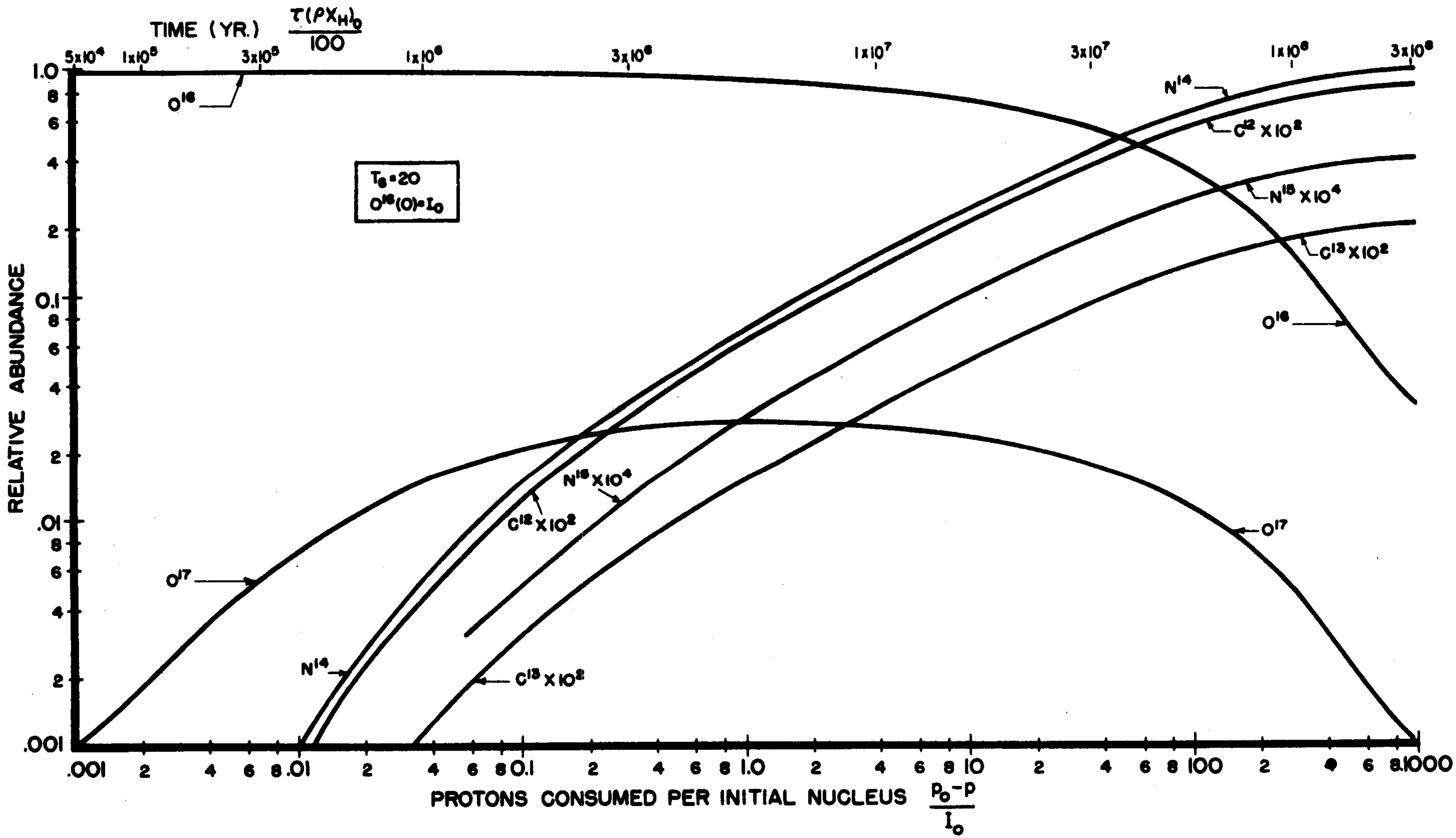


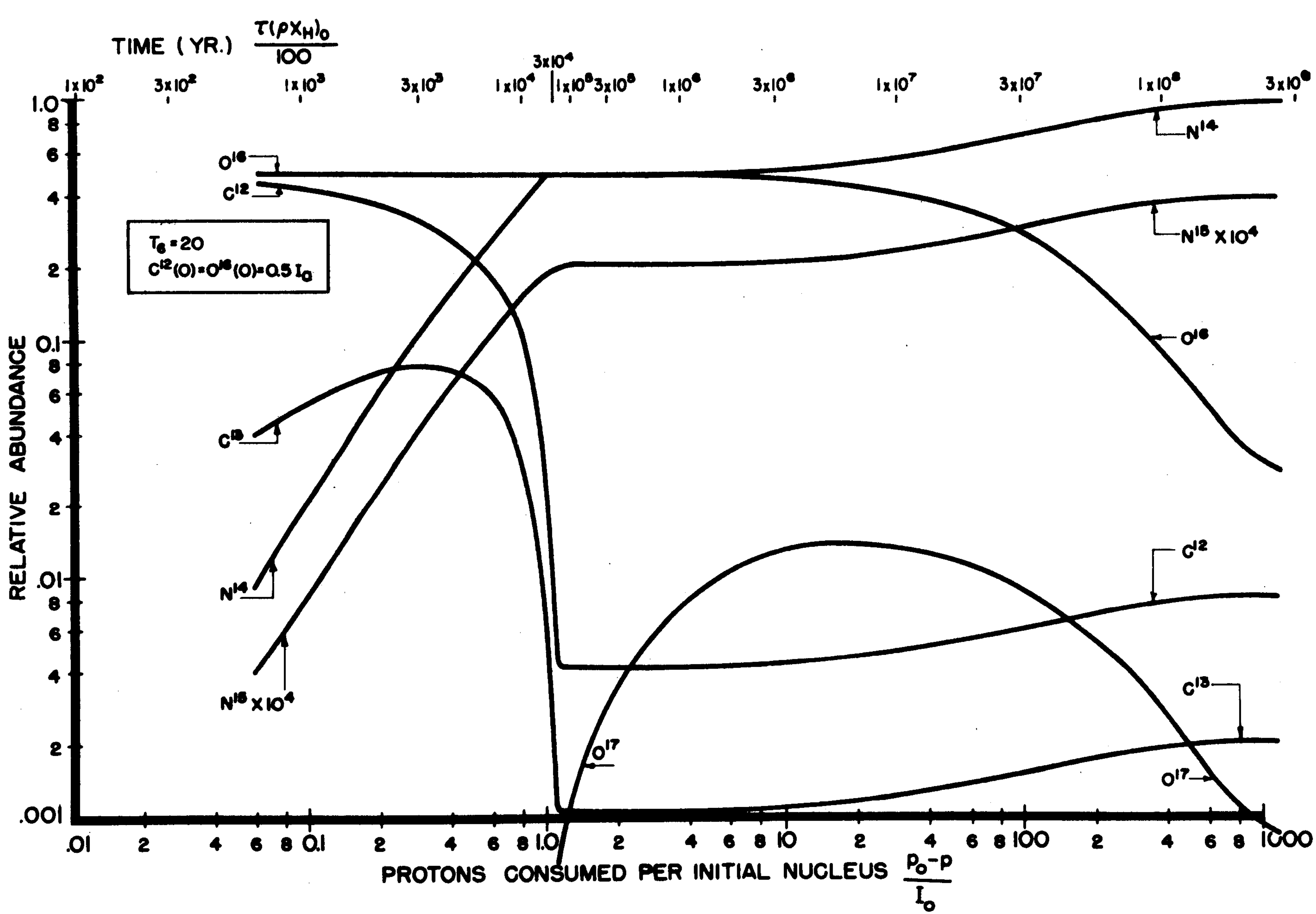
TIME (YR.) $\frac{\tau(\rho X_H)_0}{100}$
1x10^2
3x10^2
1x10^3
3x10^3
1x10^4
8x10^4
1x10^5
3x10^5
1x10^6
3x10^6
1x10^7
3x10^7
1x10^8
3x10^8
RELATIVE ABUNDANCE
1.0
0.1
.01
.001
$T_6 = 20$
$C^{12}(0) = I_0$
C^{12}
C^{13}
N^{14}
$N^{15} \times 10^4$
$O^{17} \times 10^3$
$O^{16} \times 10$
.01
0.1
1.0
10
100
1000
PROTONS CONSUMED PER INITIAL NUCLEUS $\frac{p_0 - p}{I_0}$

Fig. 1



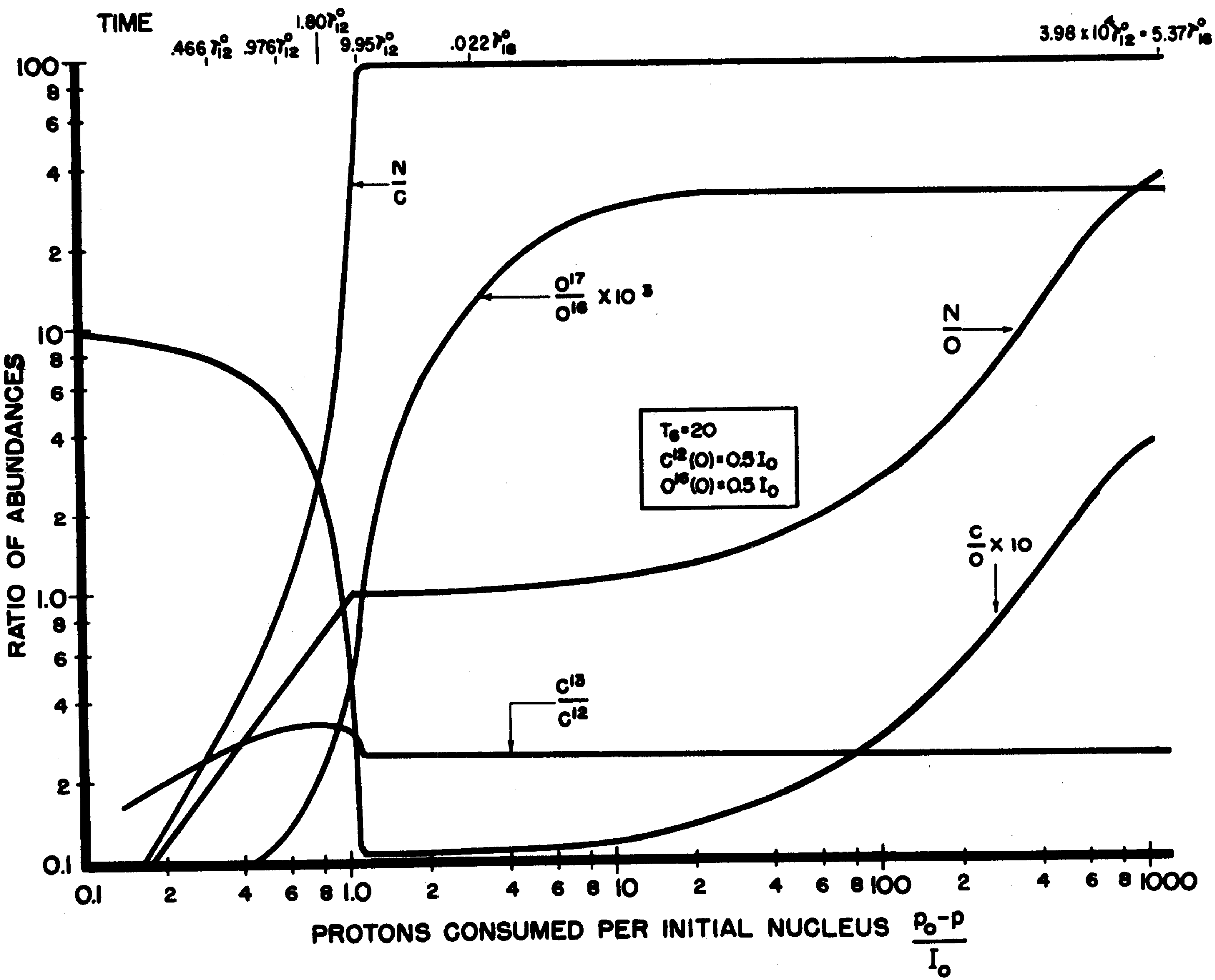
TIME (YR.) $\frac{\tau(\rho X_H)_0}{100}$
5x10⁴
1x10⁵
3x10⁵
1x10⁶
3x10⁶
1x10⁷
3x10⁷
1x10⁸
3x10⁸
RELATIVE ABUNDANCE
PROTONS CONSUMED PER INITIAL NUCLEUS $\frac{p_0 - p}{I_0}$
$T_6 = 20$
$O^{16}(0) = I_0$
O^{16}
O^{17}
N^{14}
$C^{12} \times 10^2$
$N^{15} \times 10^4$
$C^{13} \times 10^2$

Fig. 2



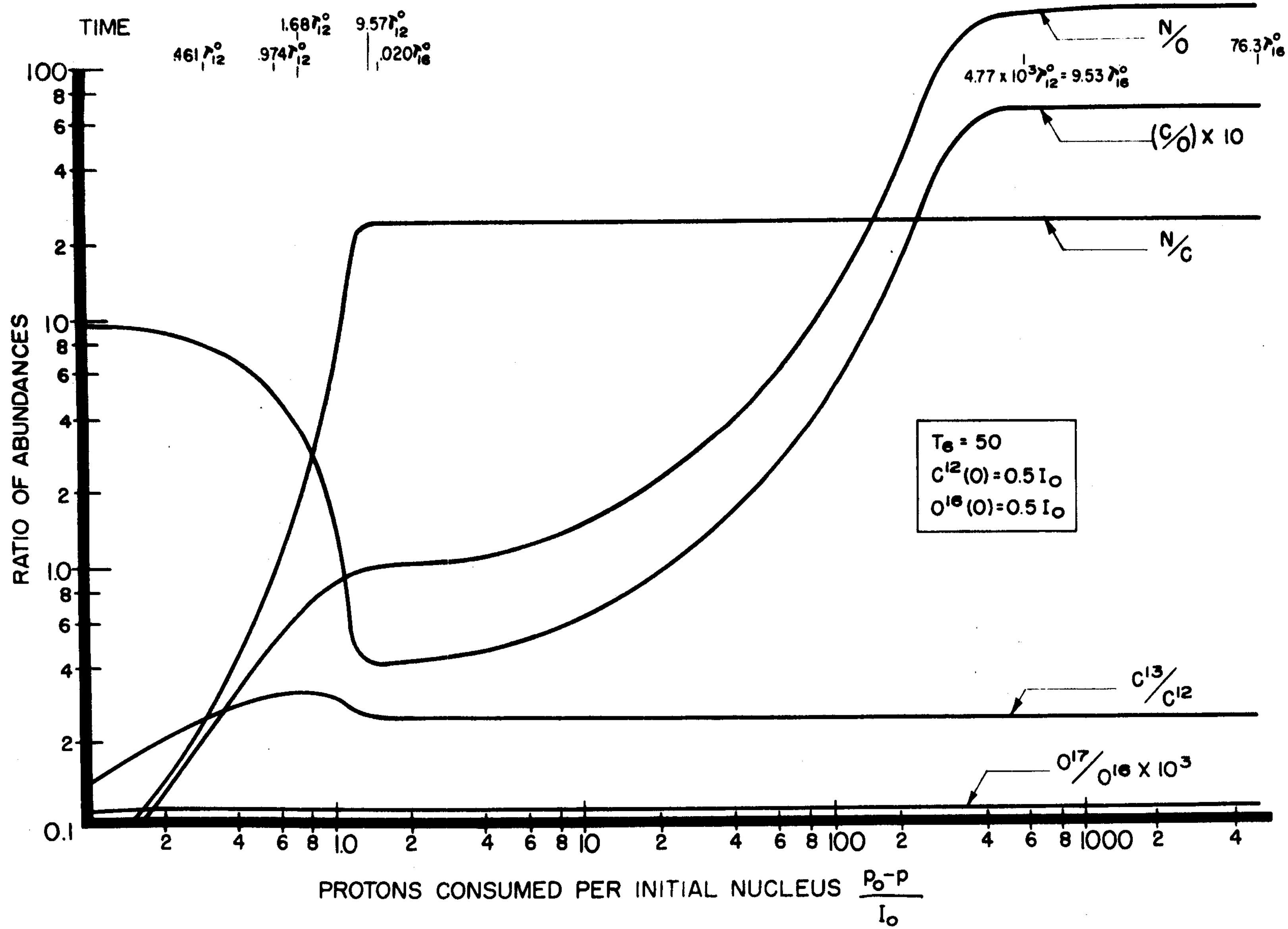
TIME (YR.) $\frac{\tau(\rho X_H)_0}{100}$
1x10^2
3x10^2
1x10^3
3x10^3
1x10^4
3x10^4
1x10^5
3x10^5
1x10^6
3x10^6
1x10^7
3x10^7
1x10^8
3x10^8
RELATIVE ABUNDANCE
1.0
0.1
.01
.001
$T_6 = 20$
$C^{12}(0) = O^{16}(0) = 0.5\ I_0$
O^{16}
C^{12}
C^{13}
N^{14}
$N^{15} \times 10^4$
O^{17}
.01
0.1
1.0
10
100
1000
PROTONS CONSUMED PER INITIAL NUCLEUS $\frac{p_0 - p}{I_0}$

Fig. 3



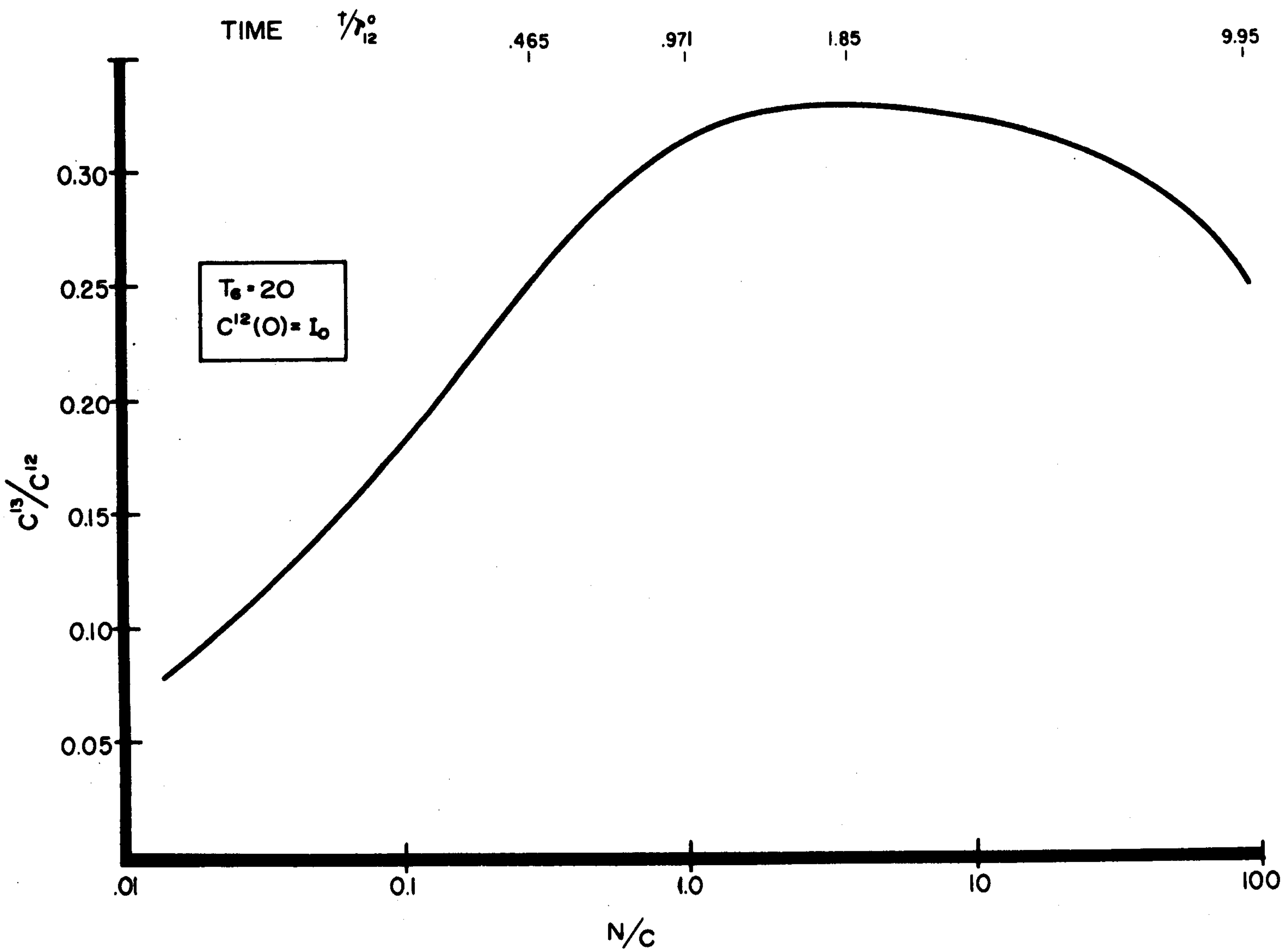
TIME
$.466\,\tau_{12}^{0}$
$.976\,\tau_{12}^{0}$
$1.80\,\tau_{12}^{0}$
$9.95\,\tau_{12}^{0}$
$.022\,\tau_{16}^{0}$
$3.98\times10^{4}\,\tau_{12}^{0}=5.37\,\tau_{16}^{0}$
RATIO OF ABUNDANCES
PROTONS CONSUMED PER INITIAL NUCLEUS $\frac{p_0-p}{I_0}$
$\frac{N}{C}$
$\frac{O^{17}}{O^{16}}\times10^{3}$
$\frac{N}{O}$
$\frac{C}{O}\times10$
$\frac{C^{13}}{C^{12}}$
$T_6=20$
$C^{12}(0)=0.5\,I_0$
$O^{16}(0)=0.5\,I_0$

Fig. 4



TIME
.461 τ^0_{12}
.974 τ^0_{12}
1.68 τ^0_{12}
9.57 τ^0_{12}
.020 τ^0_{16}
$4.77 \times 10^3 \tau^0_{12} = 9.53 \tau^0_{16}$
76.3 τ^0_{16}
N/O
(C/O) x 10
N/C
C^{13}/C^{12}
$O^{17}/O^{16} \times 10^3$
$T_6 = 50$
$C^{12}(0) = 0.5 I_o$
$O^{16}(0) = 0.5 I_o$
RATIO OF ABUNDANCES
PROTONS CONSUMED PER INITIAL NUCLEUS $\frac{p_o - p}{I_o}$

Fig. 5



TIME t/τ^0_{12}
.465
.971
1.85
9.95
$T_6 = 20$
$C^{12}(0) = I_0$
0.30
0.25
0.20
0.15
0.10
0.05
C^{13}/C^{12}
.01
0.1
1.0
10
100
N/C

Fig. 6

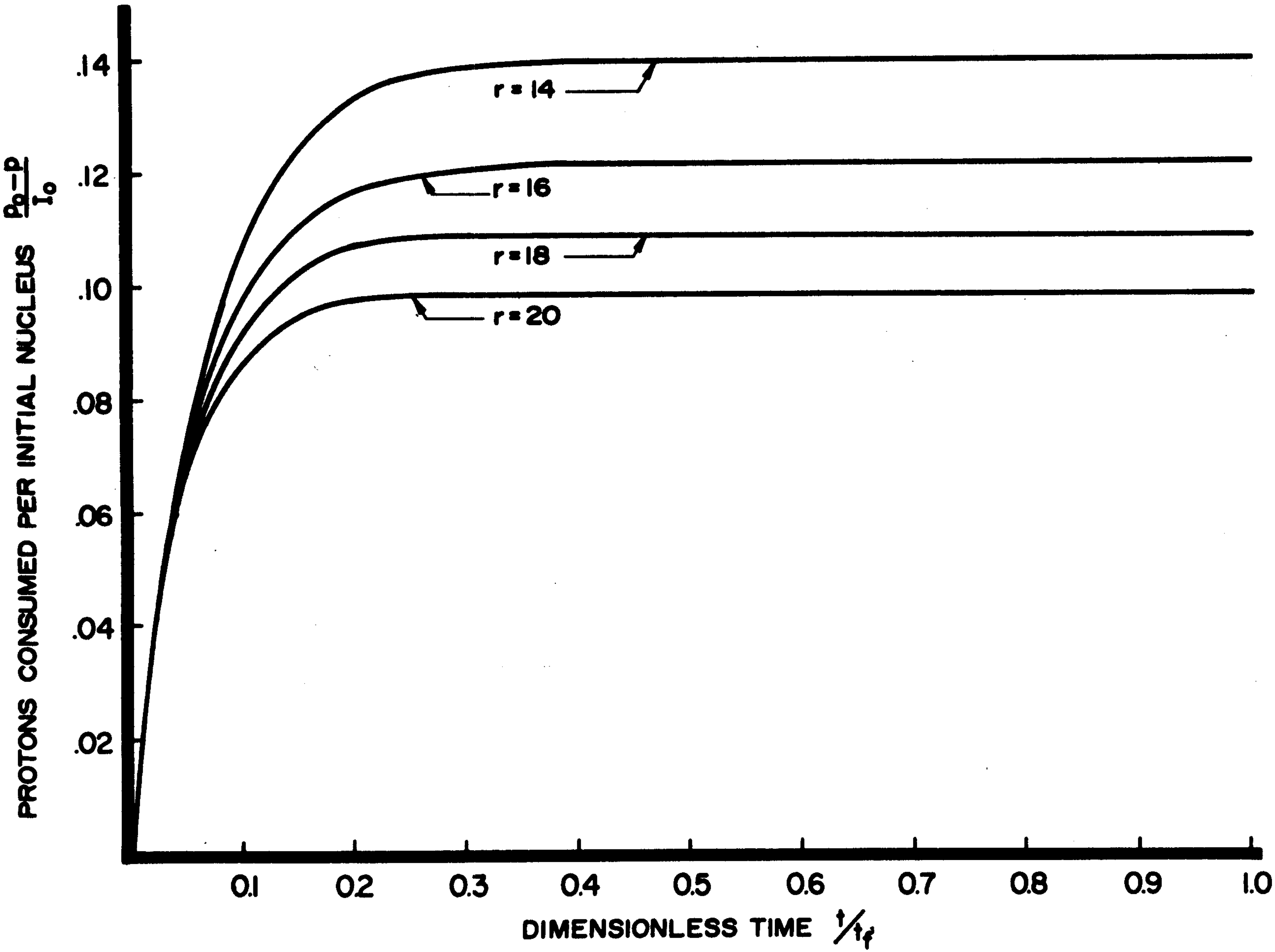

PROTONS CONSUMED PER INITIAL NUCLEUS $\frac{P_0-P}{I_0}$
.14
.12
.10
.08
.06
.04
.02
r = 14
r = 16
r = 18
r = 20
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
DIMENSIONLESS TIME t/t_f

Fig. 7